Pre Calculus **Name:** KEY
Chapter 9 Review **Block:** _____

Show all applicable work for full credit. NO credit will be given for NO work.

1. A clothing store sells pure wool and polyester-wool suits. Each suit comes in 3 colors and 10 sizes. How many suits are required for a complete assortment?

$\underset{\text{Type of suit}}{2} \cdot \underset{\text{color}}{3} \cdot \underset{\text{size}}{10} = \boxed{60}$

2. John and Kate are taking their sextuplets for a walk to the neighborhood park. The kids walk in a line holding onto six different colored rings attached to a long rope with one end for each parent. How many different ways can the family walk with the rope?

~~2 Ends~~ ~~Kate~~

$\underset{\text{Front}}{2} \cdot \underset{\text{Kids}}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \underset{\text{Back}}{1} = \boxed{240}$

3. A professor has a bank of 10 similar problems to put on a test with 3 questions. How many different tests can she design?

$\boxed{{}_{10}C_3 = 120}$

4. A small business has 21 women and 14 men employees. A committee is to be formed with four members: two women and two men. How many different committees can be formed?

$\underset{\text{Pick 2 women}}{{}_{21}C_2} \cdot \underset{\text{Pick 2 men}}{{}_{14}C_2} = 210 \cdot 91 = \boxed{19,110}$

5. How many distinguishable arrangements are in the word MATHEMATICIAN?

$\boxed{\frac{13!}{2!3!2!2!} = 129,729,600}$

2 M
3 A
2 T
2 I

M M
A A A
T T
H
E
C
I I
N

For questions 6-8, a bag contains 5 red marbles, 6 yellow marbles, and 4 green marbles. Three marbles are selected at random ***with replacement.*** ★

6. What is the probability that the second marble is green given that the first is red.

$\frac{4}{15}$

7. What is the probability that the first marble is red and the second marble is green?

$\frac{5}{15} \cdot \frac{4}{15} = \frac{4}{45} \approx .089$

8. What is the probability that exactly one of the two marbles is yellow?

YY★ ← not yellow
$P(\text{not yellow}) = \frac{9}{15}$
Each draw is independent because of replacement

${}_3C_2\left(\frac{6}{15}\right)^2 \cdot \left(\frac{9}{15}\right)^1 = 3 \cdot \left(\frac{6}{15}\right)^2 \cdot \left(\frac{9}{15}\right) = \boxed{.288}$

9. What is the probability that at least 1 of the marbles is green?

$1 - P(\text{no green}) = 1 - \left(\frac{11}{15}\right)^3$ $\quad P(\text{not green}) = \frac{11}{15}$

$= \frac{2044}{3375} \approx .606$

For questions 10-13, a standard deck of cards consists of 52 cards. Suppose two cards are drawn from the deck, but the first card is not replaced before the second card is drawn. Find the probability in each scenario.

10. Both cards are jacks. $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221} \approx .005$

11. The second card is a red 4 given that first card is an ace. $\frac{2}{51} \approx .039$

12. Exactly one card is a queen. Q ✱ ✱ Q $\frac{4}{52} \cdot \frac{48}{51} + \frac{48}{52} \cdot \frac{4}{51} = \frac{32}{221} \approx .145$

13. At least one card is a Heart. ♡♡ + ♡✱ + ✱♡

1 − ✱✱ → $1 - P(\text{no Hearts}) = 1 - \left(\frac{39}{52} \cdot \frac{38}{51}\right) = \frac{15}{34} \approx .441$

14. When there is a heavy snow (more than 6 inches), school is closed 78% of the time. When there is a light snow, school is closed 3% of the time. When there is snow, heavy snow falls 5% of the time. Find the following probabilities:

a) There is a heavy snow and school is open.

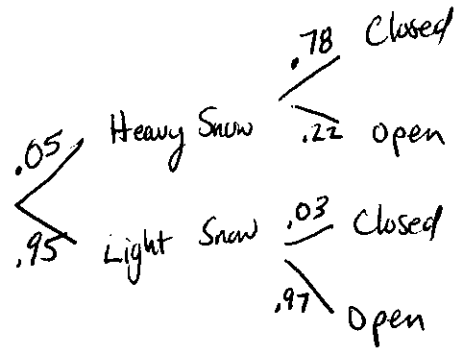
$(.05)(.22) = .011$

b) There is a light snow and school is closed.

$(.95)(.03) = .0285$

c) School gets closed.

$(.05)(.78) + (.95)(.03) = .0675$



d) There is a light snow given that school is open.

$$P(\text{Light snow} \mid \text{school's open}) = \frac{P(\text{Light snow \& school's open})}{P(\text{school's open})} = \frac{(.95)(.97)}{(.05)(.22) + (.95)(.97)} = .988$$

15. A company guarantees customer satisfaction on the purchase of a product, or the company will refund the purchase price of the product. Previous experience has shown that 10% of the purchases are returned. What is the probability that no more than 1 of the next 6 purchases will be returned?

$P(\text{return}) = .10$ $P(\text{not return}) = .9$

no more than 1 means $P(0)$ or $P(1)$ (0 returns, 1 returned) $= (.9)^6 + {}_6C_1 (.9)^5 (.1)^1 = .885735$

16. Dull Calculators, Inc. knows that a unit coming off an assembly line has a probability of 0.037 of being defective. If four units are selected at random during the course of a workday, what is the probability that none of the units are defective?

$P(\text{defective}) = .037$ $P(\text{not defective}) = .963$

$(.963)^4 = .860$

17. Expand: $(3a - 2b)^5$

$(1)(3a)^5(1) = 243a^5$
$(5)(3a)^4(-2b)^1 = 5 \cdot 81a^4 \cdot -2b = -810a^4b$
$(10)(3a)^3(-2b)^2 = 10 \cdot 27a^3 \cdot 4b^2 = 1080a^3b^2$
$(10)(3a)^2(-2b)^3 = 10 \cdot 9a^2 \cdot -8b^3 = -720a^2b^3$
$(5)(3a)^1(-2b)^4 = 5 \cdot 3a \cdot 16b^4 = 240ab^4$
$(1)(1)(-2b)^5 = -32b^5$

$$243a^5 - 810a^4b + 1080a^3b^2 - 720a^2b^3 + 240ab^4 - 32b^5$$

18. Find the coefficient of x^5y^3 in the expansion $(x+y)^8$.

$_8C_3 = 56$

19. Find the coefficient of x^4 in the expansion $(x-2)^7$.

$_7C_3\, x^4(-2)^3 = 35 \cdot x^4 \cdot -8 = -280x^4$

For questions 20-22, find the first three terms and the 10th terms of the sequence.

20. $a_k = \dfrac{(-1)^k}{k+2}$

$a_1 = \frac{-1}{3}$

$a_2 = \frac{1}{4}$

$a_3 = \frac{-1}{5}$

$a_{10} = \frac{1}{12}$

21. $a_1 = -1$

$a_n = a_{n-1} + 3$, for $n \geq 2$

$a_1 = -1$

$a_2 = 2$

$a_3 = 5$

$a_{10} = 26$

22. $b_1 = 5$

$b_n = \left(\frac{1}{2}\right) b_{n-1}$, for $n \geq 2$

$b_1 = 5$

$b_2 = 5/2$

$b_3 = 5/4$

$b_{10} = \frac{5}{2^9} = \frac{5}{512}$

For questions 23 and 24 determine whether the sequences are arithmetic or geometric. Then find (a) an explicit and (b) recursive formula for the nth terms.

23. 12, 9.5, 7, 4.5...

arithmetic $d = -2.5$

a) $a_n = 12 - 2.5(n-1)$

b) $a_1 = 12$, $a_n = a_{n-1} - 2.5$, $n \geq 2$

24. –5, –20, –80, –320...

geometric $r = 4$

a) $a_n = (-5)r^{n-1}$

b) $a_1 = -5$, $a_n = (a_{n-1}) \cdot (-5)$, $n \geq 2$

For questions 25 - 29, apply the correct formula or no credit.

25. The third and tenth terms of an arithmetic sequence are 14 and –10.5, respectively. Find the first term and *d*.

$a_3 = 14$

$a_{10} = -10.5$

$-10.5 = a_1 + 9d$

$-(\ 14 = a_1 + 2d\ )$

$-24.5 = 7d$

$-3.5 = d$

$-10.5 = a_1 + 9(-2.5)$

$21 = a_1$

$a_n = 21 - 3.5(n-1)$

26. The fourth and ninth terms of a geometric sequence are –192 and 196,608, respectively. Find the first term and r.

$a_4 = -192$

$a_9 = 196{,}608$

$196{,}608 = a_1 \cdot r^8$

$-192 = a_1 \cdot r^3$

$\rightarrow a_1 = -192/r^3 \rightarrow 196608 = \left(\frac{-192}{r^3}\right) \cdot r^8$

$196608 = -192r^5$

$-1024 = r^5$

$-4 = r$

$a_1 = \frac{-192}{4^3} = -3$

27. Find the sum of the arithmetic series: $-11 - 8 - 5 - 2 + 1 + 4 + 7 + 10$.

$S = \frac{(-11 + 10) \cdot 8}{2} = \frac{-1 \cdot 8}{2} = \boxed{-4}$

28. Find the sum of the terms of the geometric series: $4 - 2 + 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$.

$S = \frac{4\left(1 - (-1/2)^6\right)}{1 - (-1/2)} = \frac{4(1 - 1/64)}{3/2} = \boxed{\frac{21}{8}}$ $r = -1/2$

$= 2{,}625$

29. Find: $\sum_{k=1}^{45}(-4k-6)$

a_1 = when $k=1$, $-4(1)-6 = -10$

$a_{45} = -4(45) - 6 = -186$

Arithmetic series $\Rightarrow$ $S = \frac{(-10 + -186)45}{2} = \boxed{-4410}$

For questions 30 – 32, determine if the geometric series converges. If it does, find its sum.

30. $\sum_{j=1}^{\infty} 2\left(\frac{3}{4}\right)^{j-1}$ $a_1 = 2 \cdot \left(\frac{3}{4}\right)^0 = 2$

Since $r = \frac{3}{4} < 1$ the series converges.

$S = \frac{2}{1 - 3/4} = \frac{2}{1/4} = \boxed{8}$

31. $\sum_{k=1}^{\infty} 5\left(\frac{6}{7}\right)^k$ $a_1 = 5 \cdot \left(\frac{6}{7}\right)^1 = \frac{30}{7}$

Since $r = \frac{6}{7} < 1$ the series converges.

$S = \frac{30/7}{1 - 6/7} = \frac{30/7}{6/7} = \boxed{5}$

32. $\frac{1}{64} + \frac{1}{32} + \frac{1}{16} + \frac{1}{8} + \ldots$

Since $r = 2 > 1$ the series diverges

For questions 33 and 34, write the sum in sigma notation.

33. $-8 - 3 + 2 + \ldots + 92$ $a_n = 92$, Find n.

$a_1 = -8$

$d = 5$

$92 = -8 + 5(n-1)$

$100 = +5n + 5$

$95 = +5n$

$19 = n$

$\sum_{n=1}^{19} -8 + 5(n-1)$

34. $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots$

$a_1 = 1$

$r = 1/2$

$\sum_{n=1}^{\infty} 1 \cdot \left(\frac{1}{2}\right)^{n-1}$

35. Express the rational numbers as a fraction of integers. NO work = NO credit.

a) .111111…

$.1 + .01 + .001 + .0001 + \ldots$

$a_1 = .1$

$r = \frac{1}{10} = .1$ $S = \frac{.1}{1 - .1} = \frac{.1}{.9} = \boxed{1/9}$

b) 5.12121212…

$5 + .12 + .0012 + .000012 + \ldots$

$5 + S = 5 + 4/33 = 5\,4/33 = \boxed{\frac{169}{33}}$

where $S = \frac{.12}{1 - .01} = \frac{.12}{.99} = \frac{4}{33}$