Pre Calculus **Name:** Cofield
Chapter 3 Review **Block:** ______

No Calculator. Show all applicable work for full credit.

Describe how to transform the graph of ~~$g(x)=2^x$ or $h(x)=\log x$~~ the parent function h(x) *into the graphs of f(x) below. Sketch the graph by hand including 2 points and the asymptote.*

Graphs on back

parent: $h(x)=3^x$ (0,1) (1,3) HA: y=0

1. $f(x)=3^{-x}$

f(x) flips over y-axis

$(x,y)\to(-x,y)$

(0,1)
(-1,3) HA: y=0

2. $f(x)=2^{x-2}+3$ parent: $h(x)=2^x$ (0,1) (1,2) y=0

f(x) moves rt 2 and up 3

$(x,y)\to(x+2,y+3)$

(2,4)
(3,5) y=3

3. $f(x)=-\log_4 x$ parent: $h(x)=\log_4 x$ (1,0) (4,1) VA: x=0

f(x) flips over x-axis

$(x,y)\to(x,-y)$

(1,0)
(4,-1) VA: x=0

4. $f(x)=\log(1-x)-3$ parent: $h(x)=\log x$ (1,0) (10,1) x=0

$f(x)=\log(-x+1)-3$

$f(x)=\log[-(x-1)]-3$

f(x) flips over y-axis and moves right 1 & down 3

$(x,y)\to(-x+1,y-3)$ (0,-3) (-9,-2) x=1

5. Identify each of the following for each function below: domain and range, intercept(s), the asymptote, and end behavior using limit notation.

a) $f(x)=5\left(\frac{1}{2}\right)^x$

Dom: $(-\infty,\infty)$
Range: $(0,\infty)$
y-int: (0,5)
no x-int
asymp: y=0 (HA)
$\lim_{x\to\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=\infty$

b) $g(x)=\log_3 x$

Dom: $(0,\infty)$
Range: $(-\infty,\infty)$
x-int: (1,0)
no y-int
asymp: x=0 (VA)
$\lim_{x\to 0} f(x)=-\infty$ $\lim_{x\to\infty} f(x)=\infty$

State whether the function is an exponential growth function or an exponential decay function.

6. $y=e^{4-x}+2$

$e^{-x}=\left(\frac{1}{e}\right)^x<1$

exp. decay

7. $y=5\left(\frac{3}{2}\right)^x$

$\frac{3}{2}>1$

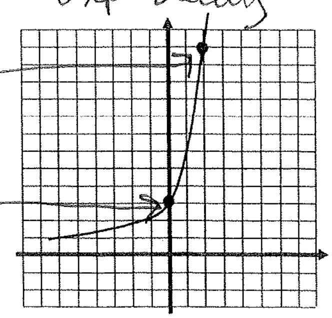
exp. growth

8. $y=3^{-2x}$

$3^{-2x}=\left(\frac{1}{3^2}\right)^x=\left(\frac{1}{9}\right)^x<1$

exp. decay

9. Find the equation of the exponential function shown at right. (2, 12) (0,3)



$y=ab^x$

$y=3\cdot b^x$

$12=3\cdot b^2$

$4=b^2$

$2=b$

$\boxed{y=3(2)^x}$

$(0,12)$ $\qquad$ $C=60$

10. Find the logistic function of the form $f(x) = \dfrac{c}{1+ab^x}$ whose initial value is 12, limit to growth is 60, and passes through (1, 24).

$$f(x) = \frac{60}{1+ab^x}$$

$$f(0) = \frac{60}{1+ab^0} = 12$$

$$60 = 12(1+a)$$

$$5 = 1+a$$

$$4 = a$$

$$f(1) = \frac{60}{1+4b^x} = 24$$

$$60 = 24(1+4b^x)$$

$$\frac{60}{24} = 1+4b^x$$

$$\frac{5}{2} = 1+4b$$

$$\frac{3}{2} = 4b$$

$$\frac{3}{8} = b$$

$$\boxed{f(x) = \frac{60}{1+4\left(\frac{3}{8}\right)^x}}$$

11. Find the (a) y-intercept and (b) the horizontal asymptotes of the function.

$$f(x) = \frac{100}{1+3(2)^x}$$

a) $f(0) = \dfrac{100}{1+3(2)^0} = \dfrac{100}{4} = 25$ $\boxed{(0,25)}$

b) $\boxed{y=0;\ y=100}$

Evaluate: Remember NO calculator.

12. $\log_{\frac{1}{81}} 3$

$\left(\frac{1}{81}\right)^x = 3$

$3^{-4x} = 3^1$

$-4x = 1$

$\boxed{x = -\frac{1}{4}}$

13. $\log_4 1$

$4^x = 1$

$\boxed{x = 0}$

14. $\ln e^4$

$e^x = e^4$

$\boxed{x = 4}$

15. $\log \sqrt[5]{10}$

$10^x = \sqrt[5]{10}$

$10^x = 10^{1/5}$

$\boxed{x = \frac{1}{5}}$

16. $\cancel{5^{\log_5}} {}^{12}$

$\boxed{12}$

17. $\log_{16} 64$

$16^x = 64$

$4^{2x} = 4^3$

$2x = 3$

$\boxed{x = 3/2}$

Graphing Calculator Allowed

For questions 18-20, choose the appropriate equation below and then solve. You should KNOW these for your test!!

$$y = ab^x \qquad A = P\left(1+\frac{r}{n}\right)^{(n \cdot t)} \qquad A = Pe^{(r \cdot t)}$$

18. Sarah's salary as an account executive is growing at a rate of 5% per year. If her initial salary is \$36,000, how long will it take her salary to double? Solve algebraically and graphically.

$y = a(1.05)^x$

$72000 = 36000(1.05)^x$

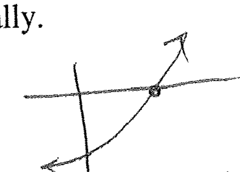
Algebraically

$2 = (1.05)^x$

$\log_{1.05} 2 = x$

$x = \dfrac{\log 2}{\log 1.05} = \boxed{14.207\text{yrs}}$

Graph



Graph $y_1 = 72000$

$y_2 = 36000(1.05)^x$

calc intersection

19. Joe invests \$1200 into an account earning 3.5% interest compounded quarterly. How much is his investment worth after 5 years?

$y = 1200\left(1+\frac{.035}{4}\right)^{(4 \cdot 5)} = \boxed{\$1428.41}$

20. The number of bacteria in an experiment are growing daily at 1.5% compounded continuously. If there were 50 bacteria present when the experiment began, how many are present on the 20th day?

$A = 50e^{(.015 \cdot 20)} = 67.492$ or $\boxed{\text{about 67 bacteria}}$

21. *Use the TVM Solver:* Find the payment made on the last day of each month for a 5 year $15000 car loan at 5.2% interest.

$n = 5 \cdot 12$ $FV = 0$ $\boxed{\$284.44}$

$I\% = 5.2$ $P/Y = 12$

$PV = 15000$ $C/Y = 12$

$PMT = ?$

22. Using 20th century US census data, the populations of New York state can be modeled by

$$P(t) = \frac{19.71}{1 + 61.22e^{-0.03563t}}$$

where P is the population in millions and t is the number of years since 1800. Based on this model,

(a) What was the population of New York in 1800? $P(0) = \frac{19.71}{1+61.22e^0} = 80.93$ or $\boxed{80{,}930{,}000 \text{ people}}$

$t = 0$

(b) What will be the population of New York in 2020?

$t = 220$ $P(220) = 19.246$ or $\boxed{19{,}246{,}000 \text{ people}}$

23. Given below is the official census population (in millions) of the state of Georgia for the years 1900-1950.

	Year	Population
0	1900	2.2
10	1910	2.6
20	1920	2.9
30	1930	2.9
40	1940	3.1
50	1950	3.4

(a) *Using your calculator,* find an exponential regression model for Georgia's population, and

$f(x) = 2.328(1.008)^x$

(b) Use the regression equation (without rounding) to predict when the population will be 5 million.

$\left.\begin{matrix} y_1 = \text{reg. equation} \\ y_2 = 5 \end{matrix}\right\}$ calc intersection 98.946 yrs after 1900 ... $\boxed{\text{during 1998}}$

24. The amount C in grams of carbon-14 present in a certain substance after t years is given by
$C = 20e^{-0.000121t}$

(a) What was the initial amount of carbon-14 present? $\boxed{20 \text{ grams}}$

(b) How much is left after 10,000 years? $\boxed{5.964 \text{ grams}}$

$t = 10{,}000$

(c) What is the half-life of carbon-14 ? Solve algebraically and graphically.

$10 = 20e^{-.000121t}$ Alg: $.5 = e^{-.000121t}$ Graph: $y_1 = 20e^{-.000121t}$

$\ln .5 = -.000121t$ $y_2 = 10$

$\frac{\ln(.5)}{-.000121} = t$ Calc intersection

$\boxed{5728.489 \text{ yrs}}$

Expand the expression. Simplify where appropriate.

25. $\log_3\left(\frac{x^3}{81y^2}\right)$

$3\log_3 x-(\log_3 81+2\log_3 y)$

$\boxed{3\log_3 x-4-2\log_3 y}$

26. $\log\left(10^{-5}z^8\cdot\sqrt[3]{x^5}\right)$

$x^{5/3}$

$-5\log 10+8\log z+\frac{5}{3}\log x$

$\boxed{-5+8\log z+5/3\log x}$

Use the properties of logarithms to write the expression as a single logarithm.

27. 4log 2 + log ½ − 3 log c

$\log(2^4\cdot\frac{1}{2})-\log c^3$

$\log 8-\log c^3=\boxed{\log\left(\frac{8}{c^3}\right)}$

28. $\underbrace{3\ln a-2\ln b-\ln a}_{\text{both in denomin.}}$

$\ln\left(\frac{a^3}{b^2 a}\right)=\boxed{\ln\left(\frac{a^2}{b^2}\right)}$

29. The relationship between intensity I of light (in lumens) at a depth of x feet in Lake Erie is given by $\log\left(\frac{I}{12}\right)=-0.00235x$. What is the intensity at a depth of 25 feet? Solve algebraically and graphically.

$\log\left(\frac{I}{12}\right)=-.00235(25)$

alg: $\log\left(\frac{I}{12}\right)=-.05875$

$10^{-.05875}=\frac{I}{12}$

$12\cdot 10^{-.05875}=I$

$\boxed{10.482 \text{ lumens}}$

graph

$y_1=-.05875$

$y_2=\log\left(\frac{x}{12}\right)$

calc. intersect.

Solve. Show all work!! Check Answers!

30. $\log_3(45)=x$

$\frac{\log 45}{\log 3}=x$

$\boxed{3.465=x}$

31. $5e^{3x}-9=28$

$5e^{3x}=37$

$e^{3x}=\frac{37}{5}$

$\ln\left(\frac{37}{5}\right)=3x$

$\frac{1}{3}\ln\left(\frac{37}{5}\right)=x$

$\boxed{.667=x}$

32. $3+5(0.6)^x=23$

$5(0.6)^x=20$

$(.6)^x=4$

$\log_{.6}4=x$

$\frac{\log 4}{\log .6}=x$

$\boxed{-2.714=x}$

33. $2\ln\left(\frac{x}{3}\right)-1=5$

+1 +1

$2\ln\left(\frac{x}{3}\right)=6$

$\ln\left(\frac{x}{3}\right)=3$

$e^3=\frac{x}{3}$

$3e^3=x$

$\boxed{60.257=x}$

34. $\log_3(1-3x)+1=5$

$\log_3(1-3x)=4$

$3^4=1-3x$

$81=1-3x$

$\boxed{\frac{80}{-3}=x}$

35. $\underbrace{\log_3(x+2)-\log_3(x-1)}=2$

$\log_3\left(\frac{x+2}{x-1}\right)=2$

$3^2=\frac{x+2}{x-1}$

$9(x-1)=x+2$

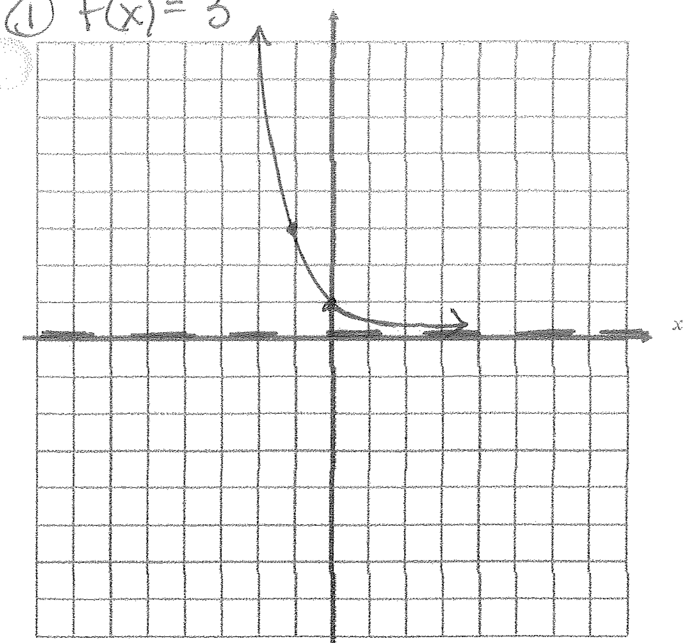
$9x-9=x+2$

$8x=11$

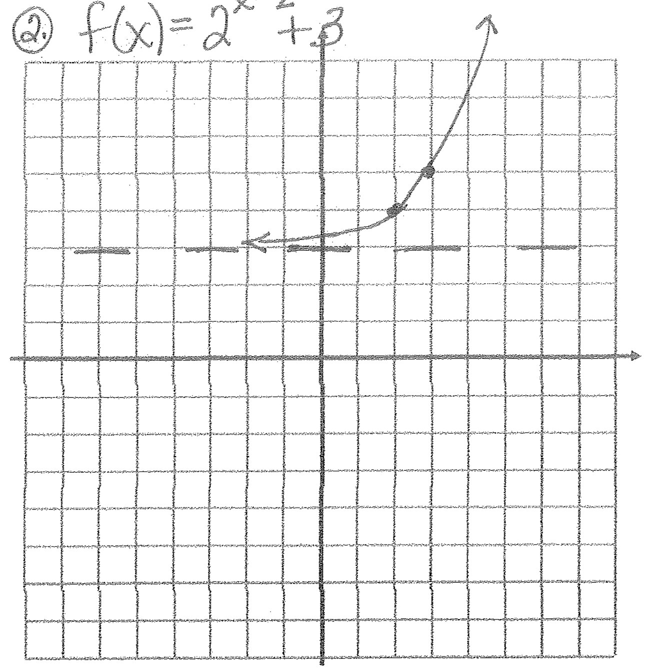
$\boxed{x=\frac{11}{8}}$

PC
Ch. 3 Review

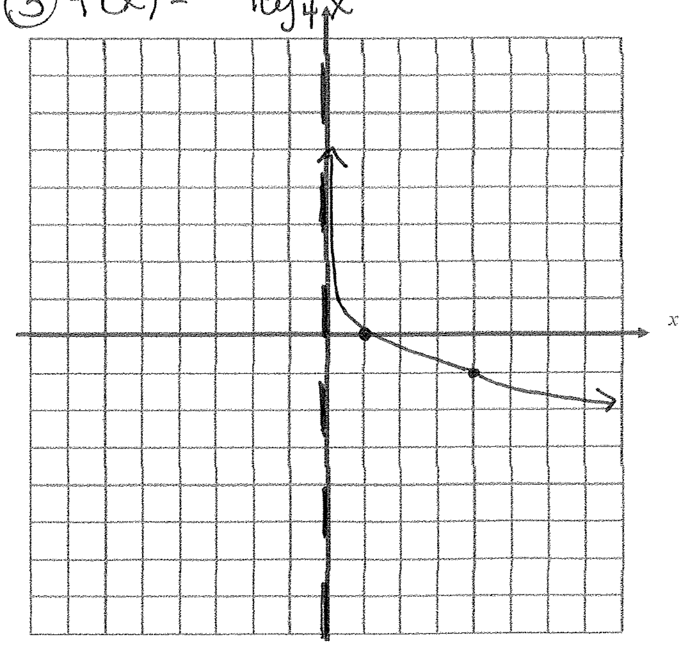
① $f(x) = 3^{-x}$



② $f(x) = 2^{x-2} + 3$



③ $f(x) = -\log_4 x$



④ $f(x) = \log(1-x) - 3$