Precalculus
Review 2.1– 2.4

Name KEY

NON CALCULATOR

Determine which are polynomial functions. For those that are, state the degree and leading coefficient.

1. $f(x) = 2x^2 + x - 9$

YES, deg = 2, leading coefficient = 2

2. $f(x) = \frac{3}{x} + 1$

NO

3. Write a linear equation for the given information $f(-2) = 5$ and $f(3) = -7$. Put your answer general form.

$(-2, 5)$ & $(3, -7)$ Slope $= \frac{5-(-7)}{-2-3} = \frac{12}{-5}$

$y - 5 = -\frac{12}{5}(x+2)$

$y - 5 = -\frac{12}{5}x - \frac{24}{5}$

$5y - 25 = -12x - 24$

$12x + 5y - 1 = 0$

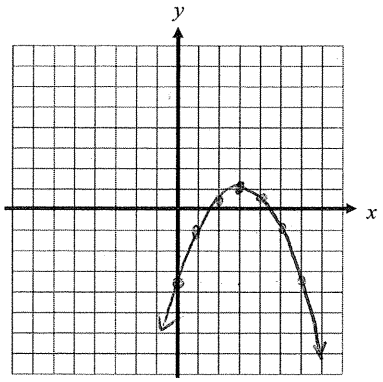
For #4 & 5 find the vertex and axis of symmetry. Sketch the graph.

4. $f(x) = -\frac{1}{2}(x-3)^2 + 1$

Axis: $x = 3$

$V(3, 1)$

$a = -\frac{1}{2}$



5. $f(x) = 2x^2 + 4x - 3$

Axis: $x = -1$

$x = \frac{-b}{2a}$

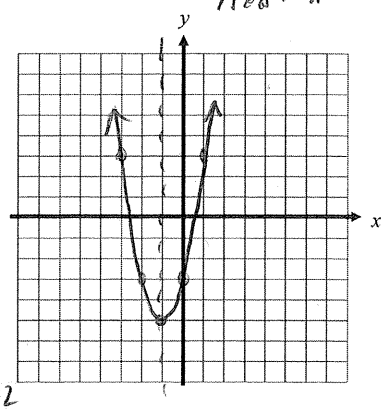
$x = \frac{-4}{2(2)} = -1$

$f(-1) = 2(-1)^2 + 4(-1) - 3$

$= 2 - 4 - 3$

$= -5$

$V(-1, -5)$ $a = 2$



6. Write an equation for the quadratic function with a vertex of (–3, 4) and containing point (–5, –8).

$y = a(x+3)^2 + 4$

$-8 = a(-5+3)^2 + 4$

$-8 = a(-2)^2 + 4$

$-8 = 4a + 4$

$-12 = 4a$

$-3 = a$

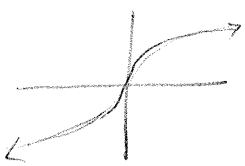
$y = -3(x+3)^2 + 4$

7. Identify the constant of variation (k) and the power (p) for the function below. Draw a sketch for each.

a) $f(x) = 4x^{1/3}$

$k = 4$

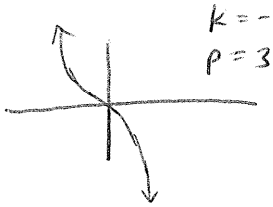
$p = 1/3$



b) $f(x) = -2x^3$

$k = -2$

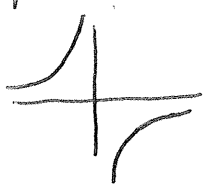
$p = 3$



c) $f(x) = -2x^{-3}$

$k = -2$

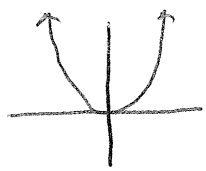
$p = -3$



d) $f(x) = (2/3)x^4$

$K = 2/3$

$p = 4$



For questions 8 and 9, write the statement as a power function equation.
Use *k* as the constant of variation.

8. H is directly proportional to the square root of t.

$H = k \cdot \sqrt{t}$ or $H = k \cdot t^{1/2}$

9. The value of Q varies inversely with the 4th power of w.

$Q = kw^{-4}$ or $Q = \frac{k}{w^4}$

For questions 10 and 11, use infinity notation to describe the end behavior.

10. $f(x) = -3x^5 + 3x^2 - 4$

as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to \infty$

11. $f(x) = 2x^4 + 3x + 7$

as $x \to \infty$, $f(x) \to \infty$

as $x \to -\infty$, $f(x) \to \infty$

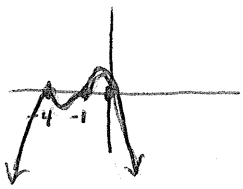
For questions 12 and 13 do the following:

a) State the degree and the zeros of the polynomial function.
b) State the multiplicity of each zero.
c) Sketch the function.

12. $f(x) = -3x(x+4)^2(x+1)^3$

a) Deg = 6

Zeros: $x = 0$ multiplicity = 1
$x = -4$ multiplicity = 2
$x = -1$ multiplicity = 3



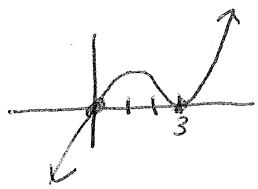
end behavior ↙ ↓ $(-3x^6)$

13. $f(x) = 3x^3 - 18x^2 + 27x$ $= 3x(x^2 - 6x + 9)$
$= 3x(x-3)(x-3)$

a) deg = 3

Zeros	multiplicity
0	1
3	2

end behavior $(3x^3)$ ↙ ↗



14. Using synthetic division, find all zeros of the function below given that –1 is a zero with multiplicity of 2.

$g(x) = 4x^4 - 4x^3 - 11x^2 + 6x + 9$

↳ it works twice...

-1	4	-4	-11	6	9
		-4	8	3	-9
-1	4	-8	-3	9	0
		-4	12	-9	
	4	-12	9	0	

Zeros	multiplicity
-1	2
3/2	2

$4x^2 - 12x + 9 = (2x-3)(2x-3) = 0$ when $x = 3/2$

15. Find a polynomial function with a leading coefficient of 2 and zeros of $\frac{1}{3}$, –1 and 4.
Express the answer in standard form.

multiply it out

$y = 2(x - 1/3)(x+1)(x-4)$

$y = (2x - 2/3)(x^2 - 3x - 4) = 2x^3 - 6x^2 - 8x - 2/3\,x^2 + 2x + 8/3$

$y = 2x^3 - 20/3\,x^2 - 6x + 8/3$

For questions 16 and 17, divide and write a summary statement in polynomial form.
Is the divisor a factor of the polynomial?(yes/no)

16. $\dfrac{4x^3 - 8x^2 + 3x + 4}{2x+1} =$

$2x^2 - 5x + 4$

$2x+1 \;\overline{)\; 4x^3 - 8x^2 + 3x + 4}$

$-(4x^3 + 2x^2)$

$-10x^2 + 3x$

$-(-10x^2 - 5x)$

$8x + 4$

$-(8x + 4)$

0

YES... since the remainder = 0 (2x+1) is a factor

$4x^3 - 8x^2 + 3x + 4 = (2x+1)(2x^2 - 5x + 4)$

17. $(2x^3 - 5x + 9) \div (x+3)$

-3	2	0	-5	9
		-6	18	-39
	2	-6	13	-30

No, (x+3) is NOT a factor since the remainder ≠ 0

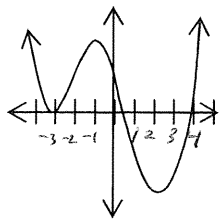
$(2x^3 - 5x + 9) = (x+3)(2x^2 - 6x + 13) - 30$

18. Use the graph below to help factor $f(x)=2x^4+3x^3-32x^2-57x+36$. $= (x-4)(x+3)^2(2x-1)$

Zeros: $x=4$

$x=-3$ (multiplicity = 2)

4	2	3	-32	-57	36
		8	44	48	-36
-3	2	11	12	-9	0
		-6	-15	9	
-3	2	5	-3	0	
		-6	3		
	2	-1	0		

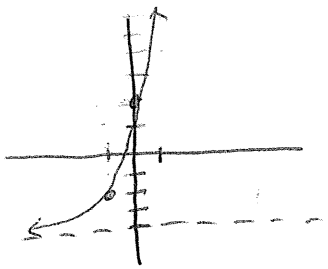


The last factor! → $(2x-1)=0$ so then $x=1/2$

19. (Chapter 1 Review) Let $f(x)=2\cdot 3^{x+1}-4$.

a) State the parent function $y=3^x$ ... $(x,y) \to (x-1, 2y-4)$

b) Graph the function. Accurately label at least 2 points and any asymptotes.



20. (Chapter 1 Review) Prove that $g(x)=\sqrt{\frac{x+7}{3}}$ and $f(x)=3x^2-7$ are inverses of each other.

Show $f(g(x)) = g(f(x)) = x$

$f(g(x)) = f\left(\sqrt{\frac{x+7}{3}}\right) = 3\left(\sqrt{\frac{x+7}{3}}\right)^2 - 7$

$= 3\left(\frac{x+7}{3}\right) - 7$

$= x+7-7$

$= x$

$g(f(x)) = g(3x^2-7)$

$= \sqrt{\frac{(3x^2-7)+7}{3}} = \sqrt{\frac{3x^2}{3}} = \sqrt{x^2} = x$

*****CALCULATOR ALLOWED*****

For questions 21 and 22, WRITE AN EQUATION, then use your equation to solve.

21. The period of vibration (P) for a pendulum varies directly as the square root of the length L. If the period of vibration is 3.5 sec when the length is 49 inches, find k, the constant of variation. Determine what the period is when L = 5.0625 inches.

$P = k\sqrt{L}$

$3.5 = k\sqrt{49}$

$\frac{1}{2} = k$

$P = \frac{1}{2}\sqrt{L}$

$P = \frac{1}{2}\sqrt{5.0625} = 1.125$ seconds

22. The gravitational attraction (A) between two masses varies inversely as the square of the distance between them. The force of attraction is 2.25 lb when the masses are 4 ft apart. Find k, the constant of variation, and determine what the attraction is when the masses are 6 ft apart.

$A = kd^{-2}$ or $A = \frac{k}{d^2}$

$2.25 = \frac{k}{4^2}$

$36 = k$

$A = \frac{36}{d^2}$

$A = \frac{36}{6^2} = 1\frac{16}{}$

23. Use the remainder theorem to find the remainder when $f(x) = x^3 - 3x + 18$ is divided by $x + 3$.
Is $x + 3$ a factor of $f(x)$?

→ Plug in (-3) for x

$f(-3) = (-3)^3 - 3(-3) + 18$
$= -27 + 9 + 18$
$= 0$

Remainder = 0
∴ Yes, $(x+3)$ is a factor of $x^3 - 3x + 18$.

24. Use the Rational Zeros Theorem to list all the potential zeros of $f(x) = 2x^3 - 7x^2 - 49x + 5$.
(±2) (±5)

$\pm \frac{1}{1}, \pm \frac{1}{2}, \pm \frac{5}{1}, \pm \frac{5}{2}$

25. Larry uses a slingshot to launch a rock straight up from a point 6 ft above level ground with an initial velocity of 170 ft/sec.
Use the fact that $s(t) = -16t^2 + v_0 t + s_0$
(s_0) (v_0)

a. Find an equation that models the height of the rock t seconds after it is launched.

$s(t) = -16t^2 + 170t + 6$

b. What is the maximum height of the rack? When will it reach that height?
Determine the answer algebraically and graphically.

→ VERTEX... $\left(-\frac{b}{2a}, \right)$

graph s(t)
Calculate maximum
x-value = when y-value = max height

$t = \frac{-170}{2(-16)} = 5.3125$ seconds

$s(5.3125) = -16(5.3125)^2 + 170(5.3125) + 6$
$= 457.5625$ feet high

c. When will the rock hit the ground? Determine the answer algebraically and graphically.
(↓)

graph s(t)
calculate zero

$s(t) = 0 = -16t^2 + 170t + 6$

$t = \frac{-170 \pm \sqrt{170^2 - 4(-16)(6)}}{2(-16)} \approx$ ~~-.035~~ or 10.660 seconds

26. The table shows the number of employees of the Gizmo Company.

	(2)	(5)	(9)	(10)	(13)	(16)
Year	~~1972~~	~~1975~~	~~1978~~	~~1980~~	~~1983~~	~~1986~~
Number of Employees	247	475	658	546	493	605

a) Find a cubic regression equation, using x = years after 1970.

$y = .834x^3 - 25.871x^2 + 248.115x - 167.677$

b) Use the regression equation to predict the number of employees in 1990. $x = 20$

Plug $x = 20$ into last equation

$y \approx 1118$ employees

27. (Chapter 1 Review) To make 10 kg of aluminum alloy with 60% aluminum, a scientist wants to use two metals with 45% and 70% aluminum content respectively. How much of each metal should she use?

45% Al (X) → 60% Al 10 kg ← 70% Al (10-X)

$.45x + .70(10 - x) = .60(10)$
$.45x + 7 - .7x = 6$
$-.25x = -1$
$x = 4$

4 kg of 45% AL
6 kg of 70% AL