Chapter 1 Review SOLUTIONS

Non-Graphing Calculator

For(#1-3) do the following:
- (a) Identify the parent
- (b) Describe the transformation.
- (c) identify the domain and range and
- (d) Sketch the graph

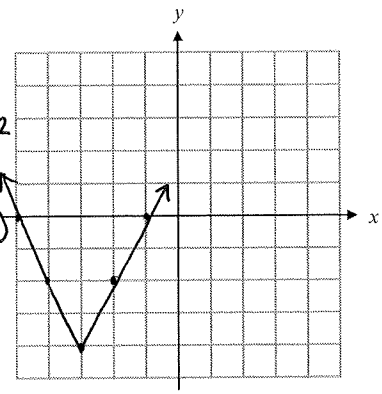
1. $y = 2|x+3| - 4$

a) $y = |x|$

b) vertical stretch x2
Left 3, Down 4

$(x,y) \to (x-3, 2x-4)$

c) D: $\mathbb{R}$
R: $y \geq -4$



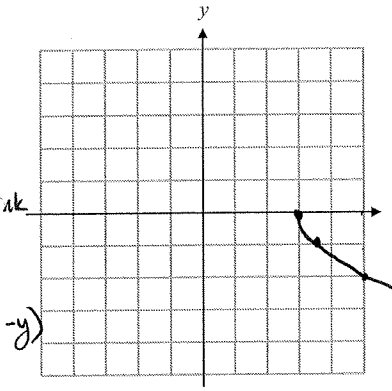
2. $y = -\sqrt{2x-6} = -\sqrt{2(x-3)}$ ← Be sure to factor out the 2 here!

a) $y = \sqrt{x}$

b) Reflect over x-axis
Horizontal shrink x ½
Right 3

$(x,y) \to \left(\frac{1}{2}x+3, -y\right)$

c) D: $x \geq 3$
R: $y \leq 0$



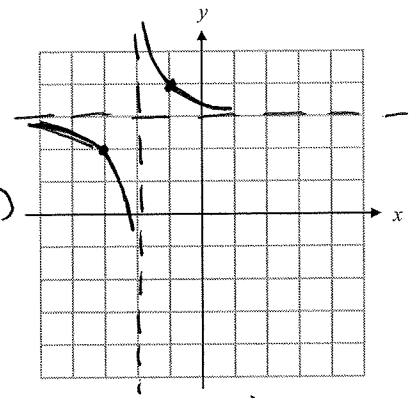
3. $y = \dfrac{1}{x+2} + 3$

a) $y = \frac{1}{x}$

b) Left 2, ~~Right~~ Up 3

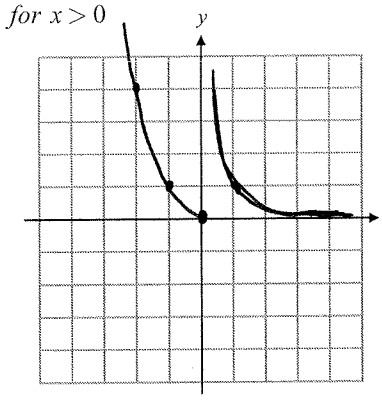
$(x,y) \to (x-2, y+3)$

c) D: $x \neq -2$
R: $y \neq 3$



... or...
D: $(-\infty,-2) \cup (-2,\infty)$ R: $(-\infty,3) \cup (3,\infty)$

4. Sketch: $f(x) = \begin{cases} x^2 \text{ for } x \leq 0 \\ \frac{1}{x} \text{ for } x > 0 \end{cases}$



Prove algebraically whether the function is even, odd, or neither.

5. $f(x) = 7x^4 - x^2$

$f(-x) = 7(-x)^4 - (-x)^2$
$= 7x^4 - x^2$
$= f(x)$
$\therefore f(x)$ is EVEN.

6. $f(x) = \dfrac{3}{4x}$

$f(-x) = \frac{3}{4(-x)} = \frac{3}{-4x}$
$= -f(x)$
$\therefore f(x)$ is ODD

★ KNOW THE 12 PARENT FUNCTIONS AND PROPERTIES … BE ABLE TO ANSWER QUESTIONS LIKE THOSE ON PAGE 109 #19 – 28.

Graphing Calculator

For 7 & 8,

(a) Find the domain
(b) determine the vertical asymptote(s) and
(c) determine the horizontal asymptotes(s).

7. $g(x)=\dfrac{4x^2+6}{x+1}$

a) Domain: $x \neq -1$ ...or... $(-\infty,-1)\cup(-1,\infty)$

b) VA $x=-1$

c) HA "NONE" $\left(\lim_{x\to\infty}\frac{4x^2}{x}=\lim_{x\to\infty}\frac{4x}{1}=+\infty\right)$

8. $h(x)=\dfrac{5x^2+2}{x^2-9}=\dfrac{5x^2+2}{(x+3)(x-3)}$

a) $x\neq 3$ & $x\neq -3$ ...or... $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

b) VA $x=3$ and $x=-3$

c) HA $y=5$ $\left(\lim_{x\to\infty}\frac{5x^2}{x^2}=\lim_{x\to\infty}5=5\right)$

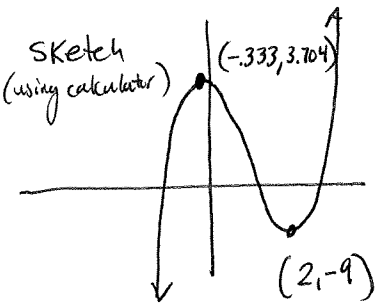
9. Graph the function and (x-values!)
(a) state the intervals on which the function is increasing, decreasing, or constant.
(b) Find all relative maximum and minimum values. (y-values!)

$f(x)=2x^3-5x^2-4x+3$

a) increasing: $(-\infty,-.333)\cup(2,\infty)$
decreasing: $(-.333,2)$

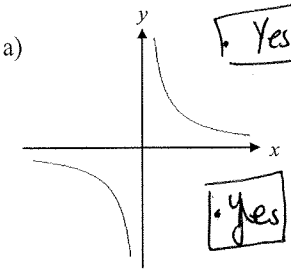
b) rel. max. = 3.704 rel. min. = −9

Sketch (using calculator)



10. Determine if the graphs are functions. Is the graph one-to-one? Explain.

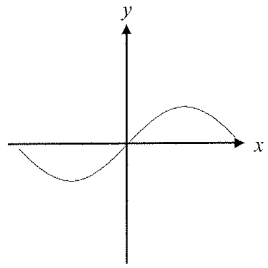
a)



Yes it is a function
it passes the vertical line test

Yes it is one-to-one ...
it passes the horizontal line test

b)



Yes it is a function,
it passes the vertical line test

No it is not one-to-one,
it does NOT pass the horizontal line test.

For #11-17, Given: $f(x)=2x^2-7$ and $g(x)=3x-2$. Find

11. $g(f(-2))$ $f(-2)=2(-2)^2-7=1$

$g(f(-2))=g(1)=3\cdot 1-2=1$

12. $f+g$

$=(2x^2-7)+(3x-2)$

$=2x^2+3x-9$

13. fg

$=(2x^2-7)(3x-2)$

$=6x^3-4x^2-21x+14$

14. $f^{-1}(x)$

$y=2x^2-7$
$x=2y^2-7$ (switch x & y)
$x+7=2y^2 \Rightarrow \frac{x+7}{2}=y^2 \Rightarrow \sqrt{\frac{x+7}{2}}=y$

15. $g(f(x))$

$g(2x^2-7)=3(2x^2-7)-2$
$=6x^2-21-2$
$=6x^2-23$

16. reflection of $f(x)$ over the x-axis.

opposite y's ... "−y"

$-f(x)=-2x^2+7$

17. reflection of $g(x)$ over the y-axis.

opposite x's ... "−x"

$g(-x)=3(-x)-2$
$=-3x-2$

18. Find the inverse of the function. VERIFY that the function is an inverse by showing $f(g(x)) = x = g(f(x))$

a) $f(x) = 3x - 2$

$y = 3x - 2$

$x = 3y - 2$

$x + 2 = 3y$

$\frac{x+2}{3} = y = f^{-1}(x)$.

verify:

$f(f^{-1}(x)) = f\left(\frac{x+2}{3}\right) = 3\left(\frac{x+2}{3}\right) - 2$

$= x + 2 - 2$

$= x$

$f^{-1}(f(x)) = f^{-1}(3x-2) = \frac{(3x-2)+2}{3}$

$= \frac{3x}{3}$

$= x$

Done!

b) $h(x) = \frac{2x+4}{5-x}$

$y = \frac{2x+4}{5-x}$

$x = \frac{2y+4}{5-y}$

$x(5-y) = 2y + 4$

$5x - xy = 2y + 4$

$-xy - 2y = 4 - 5x$

$y(-x-2) = 4 - 5x$

$y = \frac{4-5x}{-x-2}$

$h^{-1}(x) = \frac{4-5x}{-x-2}$

verification ... see next page!

19. Joe Pearlman received a 3.5% pay raise. His salary after the raise was $37,260. What was his salary before the raise?

Let x = salary before raise

$1.035x = 37,260 \Rightarrow x = 36000$

20. Sue invested $10,000, part at 3.6% annual interest and the balance at 7.8% annual interest. How much invested at each rate if a 1-year interest payment of $667.02.

1st Account: x invested @ 3.6% = $.036x$ interest

2nd Account: $10,000 - x$ invested @ 7.8% = $.078(10000 - x)$ interest

$.036x + .078(10000 - x) = 667.02$

$.036x + 780 - .078x = 667.02$

$-.042x = -112.98$

$x = 2690$

Invested $2690 @ 3.6%

$7310 @ 7.8%

Give the equation of the function whose graph is described.

21. The graph of $y = x^2$ is reflected across the x-axis and vertically stretched by a factor of 7.

$y = -7x^2$

22. The graph of $y = \sqrt{x}$ is shifted 10 units to the left. Then the graph is shifted 7 units upward.

$y = \sqrt{x+10} + 7$

23. The graph of $y = |x|$ is horizontally shrunk by a factor of 1/2.

$y = |2x|$

$$h(h^{-1}(x)) = h\left(\frac{4-5x}{-x-2}\right) = \frac{2\left(\frac{4-5x}{-x-2}\right) + 4}{5 - \left(\frac{4-5x}{-x-2}\right)}$$

$$= \frac{\frac{8-10x}{-x-2} + \frac{4}{1}\frac{(-x-2)}{(-x-2)}}{\frac{(-x-2)}{(-x-2)}\frac{5}{1} - \frac{(4-5x)}{-x-2}}$$

$$= \frac{\frac{8-10x+-4x+-8}{-x-2}}{\frac{-5x-10-4+5x}{-x-2}}$$

$$= \frac{-14x}{-14}$$

$$= x$$

$$h^{-1}(h(x)) = h^{-1}\left(\frac{2x+4}{5-x}\right) = \frac{4 - 5\left(\frac{2x+4}{5-x}\right)}{-\left(\frac{2x+4}{5-x}\right) - 2}$$

$$= \frac{\frac{(5-x)}{(5-x)}\frac{4}{1} - \frac{10x+20}{5-x}}{-\left(\frac{2x+4}{5-x}\right) - \frac{2}{1}\frac{(5-x)}{(5-x)}}$$

$$= \frac{\frac{20-4x-10x-20}{5-x}}{\frac{-2x-4-10+2x}{5-x}} = \frac{-14x}{-14} = x$$

DONE!

Pre-Requisite Review

24. Simplify the expression. Express your answer without negative exponents.

$$\frac{(x^2y^{-3})^{-2}}{x^3y^{-6}} = \frac{x^{-4}y^6}{x^3y^{-6}} = \frac{y^6y^6}{x^3x^4} = \boxed{\frac{y^{12}}{x^7}}$$

25. Find the general form equation for the line through the point (6,-4) and parallel to the line with the equation $5x + 3y = 7$.

First find slope of this line

$3y = 7 - 5x$

$y = \frac{7}{3} - \frac{5}{3}x$

Given Slope $= -5/3$

2nd Parallel means slopes are equal

Slope of new line $= -5/3$

$y + 4 = -5/3(x - 6)$

$(y + 4 = -5/3x + 10)3$

$3y + 12 = -5x + 30$

$\boxed{5x + 3y - 18 = 0}$

26. Solve by factoring:

$4x^2 - 12x = -9$

$4x^2 - 12x + 9 = 0$

$(2x - 3)(2x - 3) = 0$

$\boxed{x = 3/2}$

27. Solve algebraically:

$$\frac{x}{x+2} + \frac{2}{x-5} = \frac{14}{x^2-3x-10}$$

$(x+2)(x-5)\left[\frac{x}{x+2} + \frac{2}{x-5} = \frac{14}{(x+2)(x-5)}\right]$

$x(x-5) + 2(x+2) = 14$

$x^2 - 5x + 2x + 4 = 14$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ or $x = -2$

Both extraneous!

$\therefore$ NO SOLUTIONS!

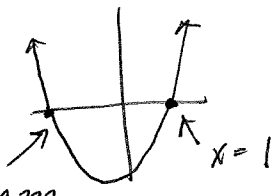
28. Solve graphically:

$3x^2 + 25x > 28$

$3x^2 + 25x - 28 > 0$

Graph in Y_1

Sketch ... calculate x-intercepts



$x = -9.333$

this parabola is above the x-axis on the interval

$\boxed{(-\infty, -9.333) \cup (1, \infty)}$