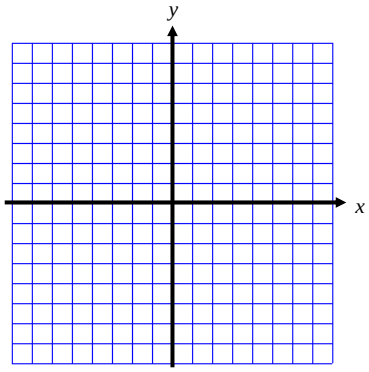
Pre-Calculus First Semester Review

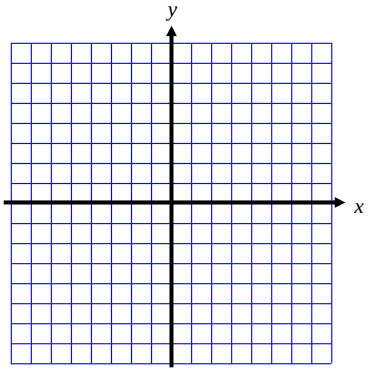
NON CALCULATOR

For question 1-6, find following:
(a) Identify the parent function.
(b) State the transformation rule or describe the transformation.
(c) Graph the function including key points and any asymptotes.

[1.5] 1. $f(x)=\dfrac{1}{x+2}-3$

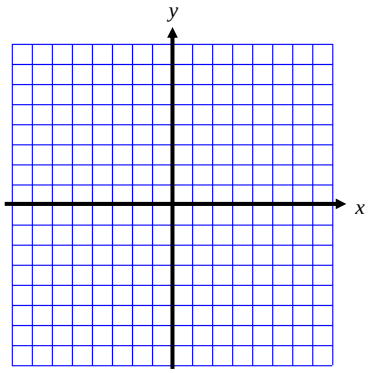


[1.5] 2. $f(x)=-3\left|2x+6\right|+1$

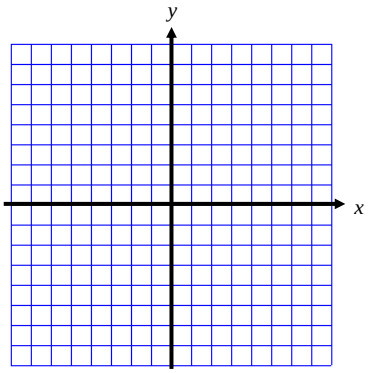


[1.5] 3. $f(x)=\log_4(x-2)$

[3.3]

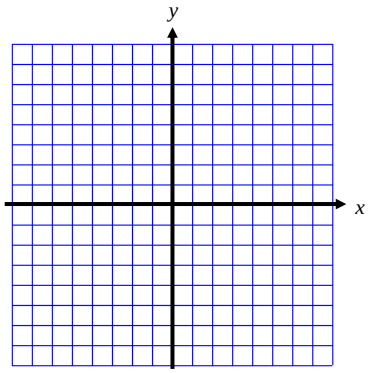


[1.5] 4. $f(x)=-\dfrac{1}{3}x^3+4$

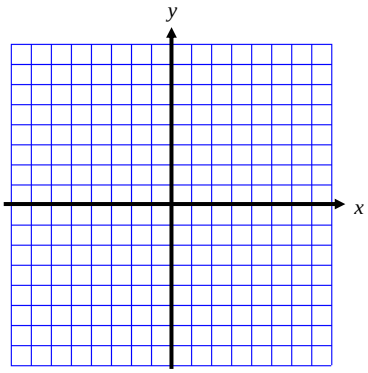


[1.5] 5. $f(x)=3^{x+2}-1$

[3.3]



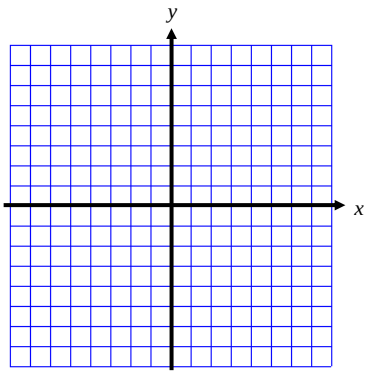
[1.5] 6. $f(x)=3\sqrt{4-x}$



For questions 7-8, find the vertex (show work). Then graph the function including at least three points.

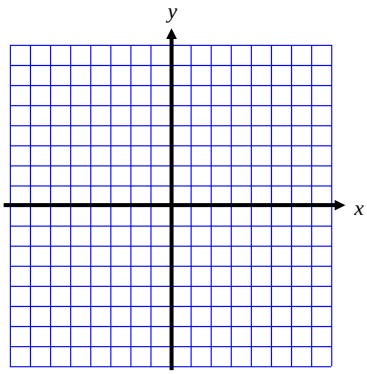
[1.5] 7. $f(x) = -2(x+1)^2 + 4$

[2.1]



[1.5] 8. $f(x) = x^2 + 8x + 11$

[2.1]



Solve. Check for extraneous roots. Write your answers in interval notation where appropriate.

[P3] 9. $2(5-2y) - 3(1-y) \geq y + 1$

[P5] 10. $-3 \leq 1 - 2x < 7$

[P5] 11. $(3x-4)^2 - 8 = 18$

[P5] 12. $|-x+4| - 3 > 7$

[P5] 13. $\dfrac{3x}{x+1} + \dfrac{5}{x-2} = \dfrac{15}{x^2 - x - 2}$

[2.8]

[P5] 14. $4x^2 - 7x + 5 = 0$

[P1] Simplify. Express your answer without negative exponents.

15. $\dfrac{\left(uv^{-2}\right)^{-3}}{u^{-5}v^{2}}$

16. $\dfrac{4x^{3}y}{x^{2}y^{3}} \cdot \dfrac{3y^{2}}{2x^{2}y^{4}}$

[P4] 17. Write the equation of a line a) parallel to **and** b) perpendicular to $5x - y = 7$ and passing through the point $(3, -4)$.

[1.2] Find the domain. Express the answer in interval notation.

18. $f(x) = \log_3(2x+5)$

19. $f(x) = \dfrac{\sqrt{7-x}}{x+4}$

[1.2] Prove algebraically whether the function is even, odd, or neither.

20. $f(x) = 3x^3 - 2x$

21. $f(x) = -2x^4 - 4x + 7$

[1.4] Given $f(x) = (x-4)^2$, $g(x) = 2x - 3$ and $h(x) = \sqrt{x+5}$. Find and simplify the answer.

22. $(f \circ h)(4)$

23. $h(g(x))$

24. $(g - f)(x)$

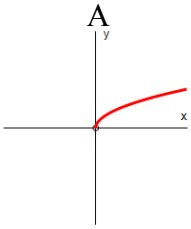
25. $(fg)(x)$

[1.4] 26. Given: $f(x) = x^3 + 2$. Find $f^{-1}(x)$.

[1.4] 27. **Verify** that f and g are inverses of each other: $f(x) = 2x + 8$ and $g(x) = \dfrac{x-8}{2}$.

[2.2] For questions 28 & 29, identify the letter of the graph below that best matches the given function.

A



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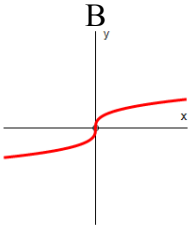

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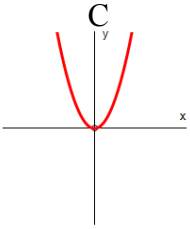

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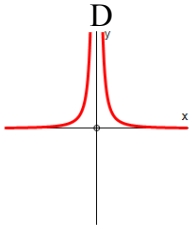

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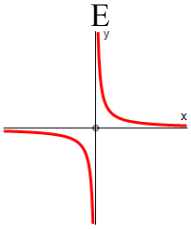

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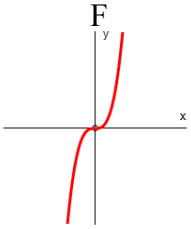


28. $f(x) = \frac{1}{2}x^{-5}$

29. $f(x) = 3x^{1/4}$

[2.3&2.7] Describe the end behavior of the polynomial or rational function using **limit** notation.

30. $f(x) = -2x^3 + 4x^2 + 1$

31. $f(x) = 3x^4 + x^2 - 5$

32. $g(x) = \dfrac{2x^3 + 6x^2 - x + 12}{x^2 + 2}$

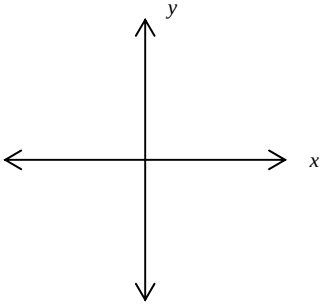
[2.3] For each function below…

a) Tell the degree.

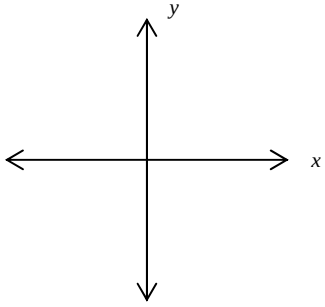
b) Find the zeros of the function with their multiplicities.

c) Sketch a graph of the function including zeros, multiplicities and end behavior.

33. $f(x) = -4(x+2)^3(2x-3)^2$



34. $f(x) = x^4 - 36x^2$



[2.5] 35. Write in $a + bi$ form: $\dfrac{2+4i}{3-2i}$

[2.6] Find the zeros of the function **and** write the function as a product of linear and irreducible quadratic factors all with real coefficients.

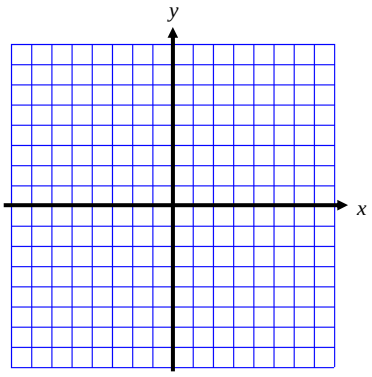
36. $f(x) = x^3 - x^2 - x - 2$, given that $x = 2$

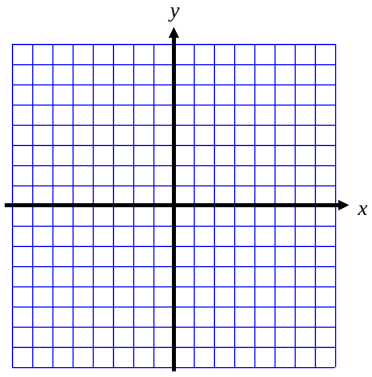
37. $f(x) = x^4 + 3x^3 - 3x^2 + 3x - 4$, given that $x = 1$ and $x = -4$

[2.7] Find (if it exists) the a) equations of any horizontal or slant asymptote, b) equations of any vertical asymptote(s) and coordinates of any holes, c) x-intercept and y-intercept, and d) graph the function including additional points in each region of the domain.

38. $g(x) = \dfrac{4x^2 - x - 5}{x^2 - 2x - 3}$

39. $g(x) = \dfrac{-2x}{x^2 - x - 6}$





[2.7] 40. Use the rational function below, along with the listed attributes, to graph the function. Include additional points in each region of the domain.

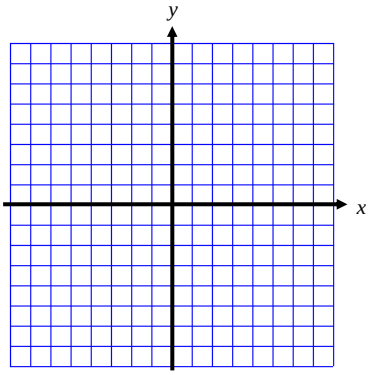
$$f(x) = \frac{x^3 + x^2 - 9x - 9}{x^2 + 2x - 3} = \frac{(x+3)(x-3)(x+1)}{(x+3)(x-1)}$$

SA: $y = x - 1$

VA: $x = 1$

x-intercepts: (3, 0) and (–1, 0)

y-intercept: (0, 3)



[2.8] Solve questions 41-43 using as sign chart.

41. $12x^3 - 14x^2 - 6x \geq 0$

42. $\frac{2x+1}{x^2+2x-3} \leq 0$

43. $\frac{2}{x+1} - \frac{3}{x-5} > 0$

[3.3] 44. Simplify

a) $\log_{11} 11^4$

b) $\log_5 1$

c) $\ln \frac{1}{e}$

d) $\log \sqrt[4]{10}$

e) $\log_9 \frac{1}{27}$

f) $3^{\log_3 7}$

[3.5] 45. Solve.

a) $\log x = -2$

b) $\log_3(2 - 3x) + 5 = 9$

c) $\log (x^2 + 21x) = 2$

d) $\log_2 (x - 1) - \log_2 (2x - 3) = 3$

Graphing Calculator Allowed

[P.5] Solve by graphing.

46. $3x - 2 = \sqrt{x+4}$

47. $0 = x^3 + x^2 - 5x + 3$

[1.2] 48. Find all a) local maxima and minima and b) identify intervals on which the function is increasing and decreasing.

$f(x) = x^3 + 2x^2 - 6x$

[1.2] Graph the function and tell whether or not it has a point of discontinuity at $x = 0$. If there is a discontinuity, tell whether it is removable or non-removable.

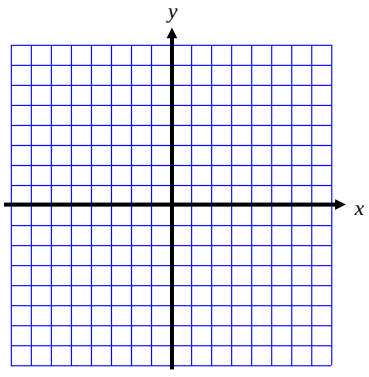
49. $f(x) = \dfrac{|x|}{x}$

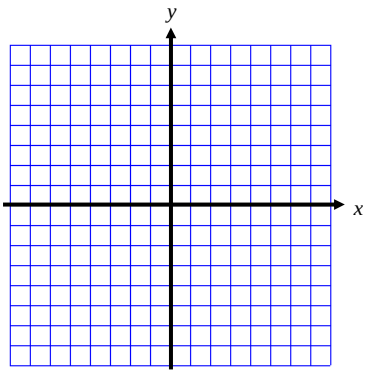
50. $h(x) = \dfrac{x^2 + x}{x}$

[1.3] Graph the piecewise-defined function. State whether the function is continuous or discontinuous at $x = 0$.

51. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$

52. $f(x) = \begin{cases} -|x| & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$





[1.3] Using the twelve basic parent functions provided in the box, list the equation of the function(s) that fit the description given.

$f(x) = x$	$f(x) = \ln x$	$f(x) = e^x$	$f(x) = x^2$	$f(x) = \|x\|$	$f(x) = x^3$
$f(x) = \sqrt{x}$	$f(x) = \frac{1}{x}$	$f(x) = \sin x$	$f(x) = \cos x$	$f(x) = \text{int}(x)$	$f(x) = \frac{1}{1+e^{-x}}$

53. Bounded (3 functions).

54. Increasing on the entire domain (6 functions).

55. Even (3 functions).

[1.6] 56. The height of a right circular cylinder equals its diameter. Write the volume of the cylinder as a function of its radius.

[1.6] 57. Sue invested $10,000, part at 3.6% annual interest and the balance 7.8% annual interest. How much is invested at each rate if a 1-year interest payment is $667.02?

[1.6] 58. The chemistry lab at the University of Hardwoods keeps two acid solutions on hand. One is 20% acid and the other is 35% acid. How much 20% acid solution and how much 35% acid solution should be used to prepare 25 liters of a 26% acid solution?

[2.1] 59. Write an equation for the linear function ***f*** with $f(-3) = -2$ and $f(4) = -8$.

[2.2] 60. Write the statement as a power function equation and answer the question. The electrical resistance of a wire varies directly as its length and inversely as the square of the diameter of the wire.

a) Write a model for this situation.

b) Suppose 50 mm of a wire of diameter 3 mm has a resistance of 8 Ω. Use this information to find the constant *k*.

c) What is the resistance of 40 mm of the same type of wire if the diameter is 4 mm?

[2.1] 61. The table below gives the weight and pulse rate of selected mammals.

a) Write a power regression equation and state the power and constant of variation.

Mammal	Body Weight	Pulse Rate (beats/min)
Rat	0.2	420
Guinea Pig	0.3	300
Rabbit	2	205
Small Dog	5	120
Large Dog	30	85
Sheep	50	70
Human	70	72

b) Use the regression equation to determine the pulse rate of a human weighing 12 pounds.

[2.4] Divide. Write a summary statement in polynomial form. Determine if the first polynomial is a factor of the second polynomial.

62. $2x + 1$; $6x^3 - 5x^2 + 9$

63. $x - 5$; $x^3 - 4x^2 - 7x + 10$

[2.4 & 2.6] Find a polynomial equation with the given zeros. Express function in standard form.

64. $\frac{1}{3}, -2, 5$ 65. $-1, 2 - i$ 66. $3, 4i$

[3.2] 67. Fruit flies are placed in a container with a banana and yeast plants. Suppose the fruit fly population after t days is given by $P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$.

a) What is the maximum number of fruit flies the container can hold?

b) How many fruit flies were originally placed in the container?

c) How long does it take for the number of fruit flies to reach one-half of the maximum flies that the container can hold?

[3.2] 68. Write the equation of the logistic function of the form $f(x) = \frac{c}{1 + ae^{-bt}}$ whose initial population is 16, limit to growth is 128 and that passes through the point (5, 32).

For questions 69-70, write a model for the situation. Be sure to clearly define your variables. Then use your model to answer the question. Solve algebraically AND graphically.

[3.2] 69. Shan invested $100 at 3.5% interest compounded monthly. How long will it take for
[3.6] her investment to double?

[3.2] 70. A radioactive isotope decays at a rate of 3% per day. A scientist has an initial amount of 50 g. Determine approximately how many days it will take for half the isotope to decay.

[3.4] 71. Rewrite the expression as a sum or difference of multiple logarithms.

a) $\log_3(a^2b)$

b) $\log_3 \dfrac{\sqrt{a}}{bc}$

[3.4] 72. Express as a single logarithm. Simplify.

a) $2\log r - \log q + 3\log w$

b) $\dfrac{1}{3}\log 27 - 2\log 4$

[3.5] 73. Solve algebraically and check graphically.

a) $2(5)^x = 26$

b) $4 + 3e^{x-5} = 157$

c) $\ln\left(\dfrac{x}{5}\right) = -0.2$

d) $5 = 21 - 2\log_3(x-7)$