6.1 VECTORS IN THE PLANE

Learning Targets:
1. Find the component form and the magnitude of a vector.
2. Perform addition and scalar multiplication of two vectors.
3. Sketch vectors and resultant vectors.
4. Use vectors to solve problems.
5. Write a vector in terms of the standard unit vectors **i** and **j**.

Definition: A *vector* is a directed line segment that has _____ and _____.
Each vector has an initial point and a terminal point.

Two vectors are _____if they have the same magnitude and direction.

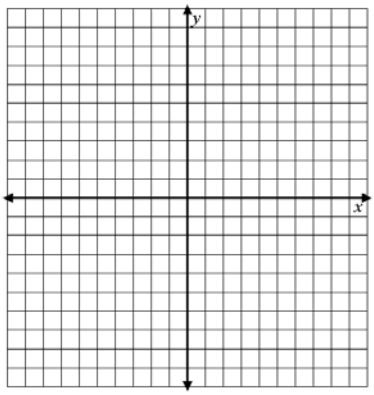
The location of the vector does not matter. A vector in *standard position* has its initial point at _____.

If vector **v** is in standard form and has a terminal point (v_1, v_2), then the *component form* can be written as _____.

The magnitude of a vector **v** is denoted _____. You can use the distance formula to find the magnitude.

When the vector is in component form $\langle a, b \rangle$, the magnitude is simply _____.

Example 1: Let P (4, 1) and Q (10, 6).



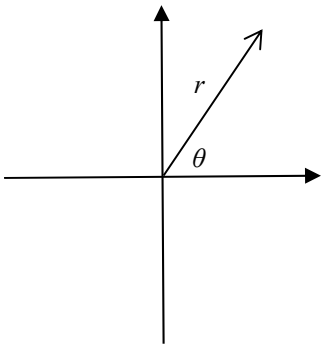
a) Sketch vector $\overrightarrow{PQ}$ and find its magnitude.

b) Draw the vector in standard position and write its component form.

c) Show algebraically why vector $\overrightarrow{PQ}$ is equivalent to the standard position vector.

A more precise way to specify the direction of a vector is to give the direction as an angle measurement.

Example 2: Consider the vector below with magnitude r and direction θ. If the endpoint of the vector is (x, y) what is the component form of the vector in terms of r and θ ?



Example 3: Sketch vector **v** in standard position with magnitude 41 and direction 220°.

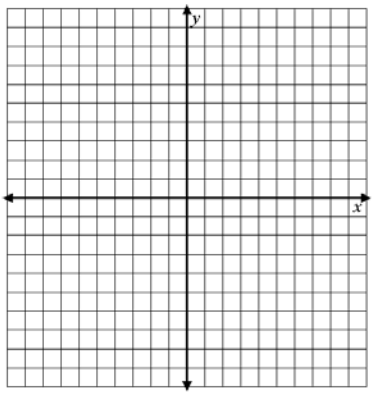
Example 4: Find the component form of the vector in the previous example.

Vector Operations

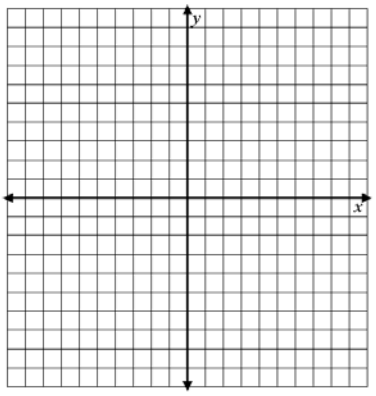
The two basic vector operations are *vector addition* and *scalar multiplication*. Scalar multiplication is multiplying a vector by a real number (aka, a "scalar"). When two vectors are added together the "answer" is called the *resultant vector.* In the paper toss game, we wanted the resultant vector to have a magnitude and a direction that would land the paper in the trash can. We had to add the velocity vector for the paper toss to the velocity vector of the wind for that to happen.

Example 5: Given **u** = $\langle 2,-3\rangle$ and **v** = $\langle -5,-2\rangle$. Find the component form of each resultant vector.

a) 2**u** + **v**



b) **u** – **v**



Example 6: Find the component form of the resultant vectors from the previous examples algebraically.

Example 7: Forces with magnitudes of 2000 Newton and 900 Newton act on a machine part at angles of 30° and –45° respectively. Find the magnitude of the resultant vector of these two forces.

Unit Vectors

A *unit vector* is a vector in the same direction but with magnitude = _____.

How do you make something with a length of 10 become a length of 1?

If the magnitude of a vector is *m* and the component form of the vector is $\langle a, b \rangle$, then the unit vector is _____.

♬: Officially, the unit vector **u** of a non-zero vector **v** is found using $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$ … in case you were wondering ☺.

Example 8: Find the unit vector of $\langle 5, 1 \rangle$.

Standard Unit Vectors

The two unit vectors **i** = $\langle 1, 0 \rangle$ and **j** = $\langle 0, 1 \rangle$ are called *standard unit vectors*. Any vector can be written as linear combination of these unit vectors. In other words, if **v** = $\langle c, d \rangle$, then **v** = c**i** + d**j**.

Example 9: Let **u** be the vector with initial point (2, –5) and terminal point (–1, 3).

a) Find the component form of vector **u**.

b) Write **u** as a linear combination of the unit vectors **i** and **j**.

Example 10: Find the unit vector for –**i** – 3**j**.

To summarize vector notation…Vectors can be written in the following ways:

1. When typed, we use a bold font … **u**
2. When written by hand, we use a "half arrow" above the vector … $\vec{u}$ or $\overrightarrow{AB}$
3. We can use component form (which implies the vector began at the origin) … $\langle c, d \rangle$
4. We can use standard unit vectors … c**i** + d**j**

6.3 Parametric Equations and Motion

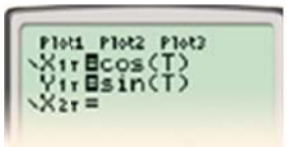
Learning Targets:

1. Graph parametric equations by hand and with calculator.
2. Eliminate the parameter to write Cartesian function.
3. Write parametric equations.
4. Solve problems involving parametric equations.

Exploration:

1. Press "**mode**" and make sure that the calculator is set to **Degree** and **Par** (for parametric mode)

2. Press "**y=**" and enter **cos(T)** for $\mathbf{X_{1T}}$ and **sin(T)** for $\mathbf{Y_{1T}}$**.** To get the variable "**T**" press the variable "**x,T,Θ,n**" button.



3. Press "**window**" and set
   Tmin = 0
   Tmax = 360
   Tstep = 2
   Xmin = –2
   Xmax = 2
   Xscl = 1
   Ymin = –2
   Ymax = 1
   Yscl = 1

4. Press "**trace".** Use the arrow keys to move the cursor left and right. What did we find? What functions made this picture? How do these functions relate to the Pythagorean identity? Can we write this function any other way?

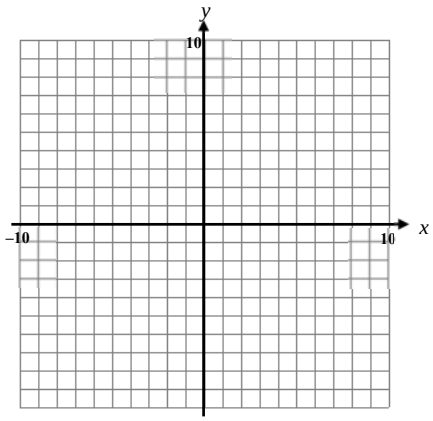
Definition: Parametric Curve

A parametric curve is the graph of ordered pairs (x, y) where

$$x = f(t)\,,\ y = f(t)$$

are functions defined on an interval of t values. Parametric equations are a set of equations where the horizontal component and the vertical component are defined according the the same variable or *parameter*.

Example 1: Create a table of values and the graph the pair of parametric equations: $x = t + 4,\ y = t^2$



Example 2: Eliminate the parameter to write a Cartesian equation to identify the graph of the curve.

a) $x = t + 4,\ y = t^2$

b) $x = 3\cos t,\ y = 3\sin t$

The easiest way to create parametric equations from an existing Cartesian equation is to simply let $x = t$ and substitute t in for every x so that y is a function of t.

Example 3: Find a parameterization for the curve $y = -16x^2 + 21x + 6$

Parametric Equations and Projectile Motion

Earlier this year, we used $h(t) = -16t^2 + v_0 t + h_0$ to model the height of an object at a given time. The graph generated by this equation is no different for an object thrown straight up than for an object thrown across the room.

We need to redefine $\begin{cases} x = t \\ y = -16t^2 + v_0 t + h_0 \end{cases}$ using a velocity vector in component form _____

A more accurate projectile motion equation is…

Example 4: A baseball is hit 129 feet per second at angle of 30° relative to level ground from an initial height of 3 feet.

a. Write the two parametric equations that model the path of the ball as a function of time.

b. After 0.5 seconds, how far has the ball traveled horizontally and vertically?

c. Determine whether the ball will clear a 10-foot wall 400 feet away.