Pre Calculus
Chapter 5 Review

Name: KEY
Block: ______

No Calculator

Simplify.

1. $\dfrac{\sin^2\beta\cot\beta}{\cos\beta}$

$= \dfrac{\sin^2\beta \cdot \frac{\cos\beta}{\sin\beta}}{\cos\beta}$

$= \sin^{\cancel{2}}\beta \cdot \dfrac{\cancel{\cos\beta}}{\cancel{\sin\beta}} \cdot \dfrac{1}{\cancel{\cos\beta}}$

$= \boxed{\sin\beta}$

2. $\sin x - \sin x\cos^2 x$

$= \sin x\,(1-\cos^2 x)$

$= \sin x\,(\sin^2 x)$

$= \boxed{\sin^3 x}$

3. $\sin^2 x + \cos\left(\dfrac{\pi}{2} - x\right) - 1 + \cos^2 x$

$\cancel{1} + \cos(\pi/2 - x) - \cancel{1}$

$\boxed{\sin(x)}$

Verify that each of the following is an identity.

4. $\dfrac{\cos^2\theta}{\sin^2\theta} + \csc\theta\sin\theta = \csc^2\theta$

$= \cot^2\theta + \dfrac{1}{\cancel{\sin\theta}}\cdot\cancel{\sin\theta}$

$= \cot^2\theta + 1$

$= \csc^2\theta$ ✓

5. $\dfrac{1}{1-\sin x} + \dfrac{1}{1+\sin x} = 2\sec^2 x$

$= \dfrac{1(1+\sin x) + 1(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \dfrac{1+\cancel{\sin x} + 1 - \cancel{\sin x}}{1-\sin^2 x}$

$= \dfrac{2}{\cos^2 x} = 2\sec^2 x$ ✓

6. $\dfrac{1+\cos 2\alpha}{\sin 2\alpha} = \cot\alpha$

$= \dfrac{\cancel{1} + (2\cos^2\alpha - \cancel{1})}{2\sin\alpha\cos\alpha}$

$= \dfrac{\cancel{2}\cos^{\cancel{2}}\alpha}{\cancel{2}\sin\alpha\cancel{\cos\alpha}}$

$= \dfrac{\cos\alpha}{\sin\alpha}$

$= \cot\alpha$ ✓

7. $\dfrac{\cos x}{1-\sin x} - \dfrac{\sin x}{\cos x} = \sec x$

$= \dfrac{\cos x(\cos x) - \sin x(1-\sin x)}{(1-\sin x)(\cos x)}$

$= \dfrac{\cos^2 x - \sin x + \sin^2 x}{(1-\sin x)(\cos x)}$

$= \dfrac{\cancel{1-\sin x}}{(\cancel{1-\sin x})(\cos x)}$

$= \dfrac{1}{\cos x} = \sec x$ ✓

Use the sum or difference identities to find the exact value of each function.

8. $\sin 105°$

$\sin(45+60)$

$\sin(45)\cos(60) + \cos(45)\sin(60)$

$\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$\boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$

9. $\tan\left(-\frac{\pi}{12}\right) = \tan(-15°)$

$= \tan(45-60)$

$= \frac{\tan(45) - \tan(60)}{1 + \tan(45)\tan(60)}$

$= \frac{1-\sqrt{3}}{1+1\cdot\sqrt{3}} = \boxed{\frac{1-\sqrt{3}}{1+\sqrt{3}}} \cdot \frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{1-2\sqrt{3}+3}{1-3} = \frac{4-2\sqrt{3}}{-2} = \boxed{-2+\sqrt{3}}$

10. $\cos 10°\cos 20° - \sin 10°\sin 20°$

$= \cos(10+20)$

$= \cos(30)$

$\boxed{= \sqrt{3}/2}$

Use the sum and difference identities to verify that each of the following is an identity.

11. $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$= \cos(\pi/2)\cos(x) + \sin(\pi/2)\sin(x)$

$= \cancel{0 \cdot \cos(x)} + 1 \cdot \sin x$

$= \sin x$ ✓

12. $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$

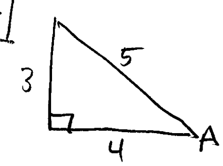
$\sin(3\pi/2)\cos(x) + \cos(3\pi/2)\sin(x)$

$= -1 \cdot \cos x + \cancel{0 \cdot \sin x}$

$= -\cos x$ ✓

If $\boxed{\sin A = \frac{3}{5}}$ *and A is in the second quadrant, find each value.*

cosine & tangent are negative $\boxed{\cos A = -\frac{4}{5} \quad \tan A = -\frac{3}{4}}$



13. cos 2A

$= 1 - 2\sin^2 A$

$= 1 - 2(\sin A)^2$

$= 1 - 2\left(\frac{3}{5}\right)^2$

$= 1 - 2 \cdot \frac{9}{25}$

$= 1 - \frac{18}{25} = \boxed{7/25}$

14. sin 2A

$= 2\sin A\cos A$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= \boxed{\frac{-24}{25}}$

15. tan 2A

$= \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan A}{1-(\tan A)^2}$

$= \frac{2(-3/4)}{1-(-3/4)^2} = \frac{-3/2}{1-\frac{9}{16}} = \frac{-3/2}{7/16}$

$= \frac{-3}{2} \cdot \frac{16}{7}$

$\boxed{= \frac{-24}{7}}$

Use double angle identities to write each of the following as the function of one angle. Simplify your answer.

16. $1 - 2\sin^2\left(\frac{\pi}{8}\right)$

$= \cos(2 \cdot \pi/8)$

$= \cos(\pi/4)$

$\boxed{= \frac{\sqrt{2}}{2}}$

17. $\frac{2\tan 75°}{1-\tan^2 75°}$

$= \tan(2 \cdot 75)$

$= \tan(150°)$

$= \boxed{-\frac{1}{\sqrt{3}}}$

For questions 18–21, solve each equation for $[0, 2\pi)$.

18. $\sqrt{2}\cos x \sin x - \sin x = 0$

$\sin x\,(\sqrt{2}\cos x - 1) = 0$

$\sin x = 0$ or $\sqrt{2}\cos x - 1 = 0$

$\boxed{x = 0 \text{ or } \pi}$ OR $\cos x = \frac{1}{\sqrt{2}}$

$\boxed{x = \pi/4 \text{ or } 7\pi/4}$

19. $\cos^2 x + 4\sin x + 4 = 0$

$(1 - \sin^2 x) + 4\sin x + 4 = 0$

$-\sin^2 x + 4\sin x + 5 = 0$

$(\sin x + 1)(-\sin x + 5) = 0$

$\sin x + 1 = 0$ or $-\sin x + 5 = 0$

$\sin x = -1$ $\quad \sin x = 5$

$\boxed{x = 3\pi/2}$ $\quad$ not possible

20. $\cos 2x - 2\sin^2 x = 0$

$(1 - 2\sin^2 x) - 2\sin^2 x = 0$

$1 - 4\sin^2 x = 0$

$-4\sin^2 x = -1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$\boxed{x = \pi/6,\ 5\pi/6,\ 7\pi/6, \text{ or } 11\pi/6}$

21. $4\sin^2 x - 3 = 0$

$4\sin^2 x = 3$

$\sin^2 x = 3/4$

$\sin x = \pm\sqrt{3}/2$

$\boxed{x = \pi/3,\ 2\pi/3,\ 4\pi/3,\ 5\pi/3}$

*For question 22–23, solve each equation for **all** values.*

22. $2\tan x \sin x + \tan x = 0$

$\tan x\,(2\sin x + 1) = 0$

$\tan x = 0$ or $2\sin x + 1 = 0$

$\sin x = -1/2$

$\tan x = 0$

when

$\sin x = 0$

$x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

$x = 0$ or π

$x = 0 + 2\pi k$
$x = \pi + 2\pi k$
$x = \frac{7\pi}{6} + 2\pi k$
$x = 11\pi/6 + 2\pi k$
, where k is an integer

23. $\cos\left(\frac{3\pi}{2} - x\right) = 1$

$\cos(3\pi/2)\cos(x) + \sin(3\pi/2)\sin(x) = 1$

$0 \cdot \cos x + -1 \cdot \sin x = 1$

$-\sin x = 1$

$\sin x = -1$

$x = 3\pi/2$

$x = \frac{3\pi}{2} + 2\pi k$
where k is an integer