

Pre Calculus  
Review 4B (4.4-4.8)

Name: \_\_\_\_\_ Key  
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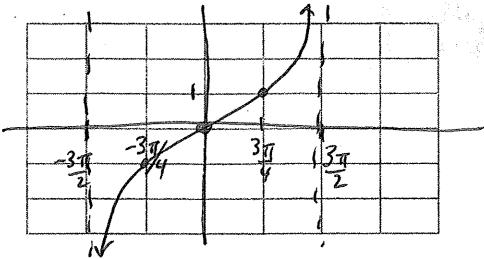
*Non-Calculator*

Now all work for credit!!

Graph each function without a calculator. State the (a) amplitude, (b) period, (c) phase shift and (d) vertical shift. Label each axis.

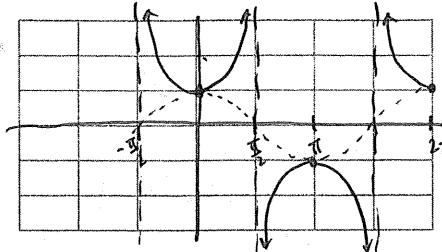
1.  $y = \tan\left(\frac{x}{3}\right)$

- a)  $\text{N/A}$
- b) Period =  $3\pi$
- c) no p.s.
- d) no v.s.



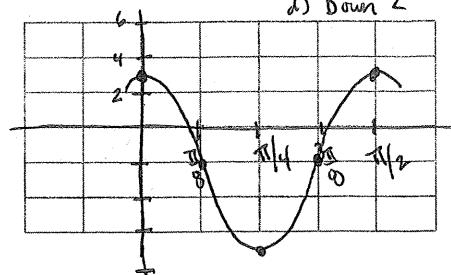
2.  $y = \sec x$

- a)  $\text{N/A}$
- b) Per =  $2\pi$
- c) no p.s.
- d) no v.s.



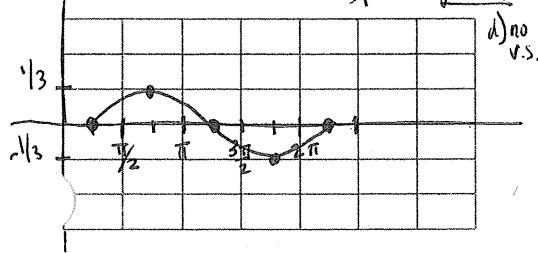
3.  $y = 5 \cos(4x) - 2$

- a) amp = 5
- b) per =  $\frac{\pi}{2}$
- c) no p.s.
- d) Down 2



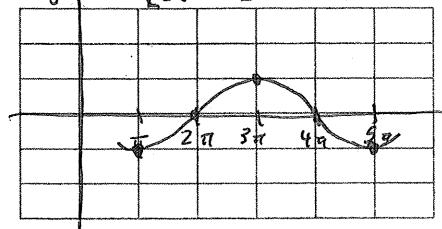
4.  $y = \frac{1}{3} \sin\left(x - \frac{\pi}{4}\right)$

- a) amp =  $\frac{1}{3}$
- b) per =  $2\pi$
- c) p.s.: Right  $\frac{\pi}{4}$
- d) no v.s.



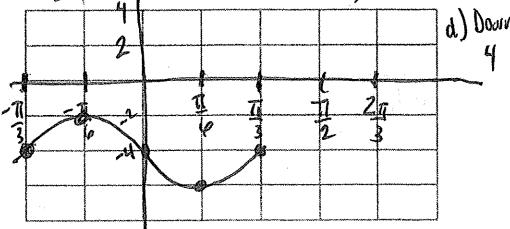
5.  $y = -\cos\left(\frac{x}{2} - \frac{\pi}{2}\right)$

- a) amp = 1 (flipped)
- b) per =  $4\pi$
- c) Right  $\pi$
- d) no v.s.



6.  $y = 2 \sin(3x + \pi) - 4$

- a) amp = 2
- b) per =  $\frac{2\pi}{3}$
- c) Left  $\pi/3$
- d) Down 4



7. Write the equation(s) of the cosine function with amplitude 5, period  $3\pi$ , phase shift  $-\frac{\pi}{6}$ , and vertical shift of 1.

$$\begin{aligned} P &= \frac{2\pi}{b} \\ 3\pi &= \frac{2\pi}{b} \\ b &= 2/3 \end{aligned}$$

$$y = 5 \cos\left[\frac{2}{3}(x + \pi/6)\right] + 1$$

Left  $\pi/6$

8. Write the equation of the function shown below using the given parent function...

a)  $y = \sin x$

$$\begin{aligned} \text{amp} &= 3 \\ \text{per} &= \pi/4 = \frac{2\pi}{b} \\ b &= 8 \end{aligned}$$

$$\begin{aligned} &8 = b \\ &\text{Left } \pi/16 \end{aligned}$$

$$y = 3 \sin\left[8\left(x + \frac{\pi}{16}\right)\right]$$

b)  $y = \cos x$

$$\begin{aligned} \text{amp} &= 6 \\ \text{down} &3 \end{aligned}$$

$$\begin{aligned} \text{per} &= 6\pi = \frac{2\pi}{b} \\ \frac{1}{3} &= b \end{aligned}$$

$$\text{Left } \frac{3\pi}{2}$$

$$y = 6 \cos\left[\frac{1}{3}(x + \frac{3\pi}{2})\right] - 3$$

c)  $y = \tan x$

$$\text{vertical shrink by } \frac{1}{2}$$

$$\begin{aligned} \text{period} &= 2\pi = \frac{\pi}{b} \\ \frac{1}{2} &= b \end{aligned}$$

$$\text{Down 1}$$

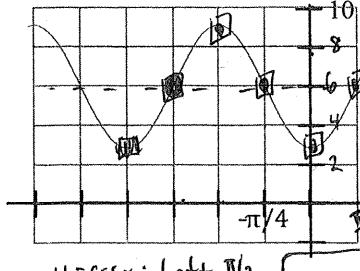
$$y = \frac{1}{2} \tan\left(\frac{1}{2}x\right) - 1$$

d)  $y = \sin x$  and  $y = \cos x$

$$\begin{aligned} \text{amp} &= 3 \\ \text{per} &= \pi = \frac{2\pi}{b} \\ 2 &= b \end{aligned}$$

$$y = \sin x : \text{Left } \pi/4 \text{ (flipped)}$$

$$y = -3 \sin\left[2\left(x + \frac{\pi}{4}\right)\right] + 6$$

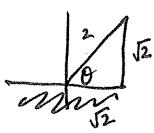


$$y = \cos x : \text{Left } \pi/2$$

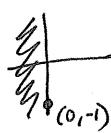
$$y = 3 \cos\left[2(x + \pi/2)\right] + 6$$

Find the exact value. Express your angle answers in radian measure.

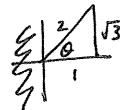
9.  $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$



10.  $\sin^{-1}(-1) = -\frac{\pi}{2}$



11.  $\arctan(\sqrt{3}) = \frac{\pi}{3}$



12.  $\sin\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

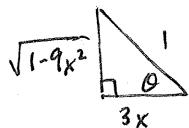
13.  $\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] = \frac{4\pi}{3}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

14.  $\arcsin\left[\tan\left(\frac{3\pi}{4}\right)\right] = \arcsin(-1)$

$-\frac{\pi}{2}$

15. Find an algebraic expression equivalent to  $\sin(\underbrace{\arccos 3x})$ .

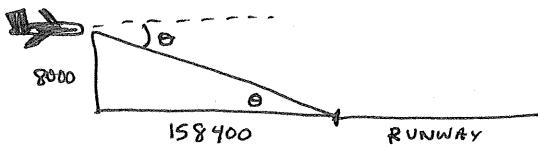


$= \sin(\theta)$

$= \sqrt{1-9x^2}$

*Calculator Allowed*

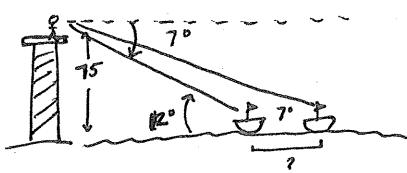
16. A plane is 8000 feet above the ground when it begins its final approach to a runway. If the ground distance to the end of the runway is 158400 feet, what is the angle of descent to the end of the runway?



$\tan \theta = \frac{8000}{158400}$

$\theta = \tan^{-1}\left(\frac{8000}{158400}\right) \approx 2.891^\circ$

17. Two boats are observed from a tower 75 meters above a lake. The angles of depression are  $12^\circ$  and  $7^\circ$ . How far apart are the boats?

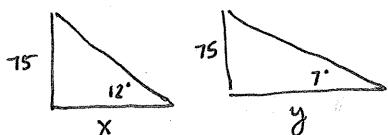


$\tan 12^\circ = \frac{75}{x}$

$x = \frac{75}{\tan 12^\circ}$

$\tan 7^\circ = \frac{75}{y}$

$y = \frac{75}{\tan 7^\circ}$



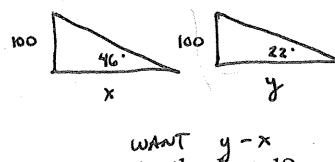
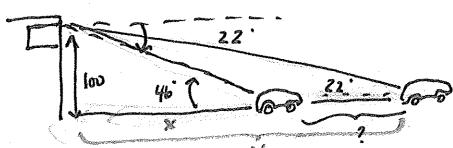
$\frac{75}{\tan 12^\circ} - \frac{75}{\tan 7^\circ} \approx 257.9787239$

Want  $y - x$

$\approx 257.979 \text{ m apart}$

18. From the Penthouse Suite in 100 ft tall hotel, a woman observes a car speeding recklessly away from the entrance to the hotel. If the angle of depression from the woman to the car changes from  $46^\circ$  to  $22^\circ$  while she is observing, find each of the following...

a) How far did the car travel away while the woman was watching?



$$\tan 46^\circ = \frac{100}{x} \quad \tan 22^\circ = \frac{100}{y}$$

$$x = \frac{100}{\tan 46^\circ} \quad y = \frac{100}{\tan 22^\circ}$$

b) How far was the car from the entrance to the hotel?

$$y - x = \frac{100}{\tan 22^\circ} - \frac{100}{\tan 46^\circ} \approx 150.9398079$$

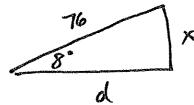
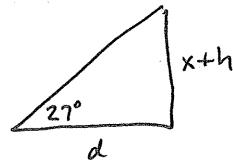
$\approx 150.940$  feet

~~at beginning~~

$$\text{at beginning} \dots x \approx \frac{100}{\tan 46^\circ} \approx 96.569 \text{ feet}$$

$$\text{at end} \dots y \approx \frac{100}{\tan 22^\circ} \approx 247.509 \text{ feet}$$

19. A certain tree grows vertically on a hill which makes an angle of  $8^\circ$  with the horizontal. When the angle of elevation of the sun is  $27^\circ$ , the end of the tree's shadow is 76 meters directly downhill from the base of the tree. Find the height of the tree.



$$\cos 8^\circ = \frac{d}{76} \quad \sin 8^\circ = \frac{x}{76}$$

$$76 \cos 8^\circ = d \quad 76 \sin 8^\circ = x$$

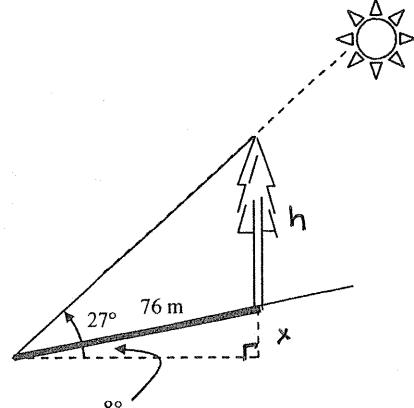
$$\tan 27^\circ = \frac{x+h}{76 \cos 8^\circ}$$

$$\tan 27^\circ = \frac{76 \sin 8^\circ + h}{76 \cos 8^\circ}$$

$$76 \cos 8^\circ \cdot \tan 27^\circ = 76 \sin 8^\circ + h$$

$$76 \cos 8^\circ \cdot \tan 27^\circ - 76 \sin 8^\circ = h$$

$$27.76991982 \approx h$$



Tree  $\approx 27.770$  feet

20. The Ferris wheel shown makes one complete turn every 30 seconds. A rider's height,  $h$ , above the ground can be modeled by a trigonometric equation

$h = a \sin(\omega t) + k$ , where  $h$  and  $k$  are given in feet and  $t$  is in seconds. Write the equation.

$$k = 33 \quad \text{Per} = 30 \text{ sec} = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{30} = \pi/15$$

$$\text{amp} = 28$$

$$h = 28 \sin\left(\frac{\pi}{15}t\right) + 33$$

