2.2 POWER FUNCTIONS WITH MODELING

Learning Targets:
1. Identify a power functions.
2. Model power functions using the regression capabilities of your calculator.
3. Understand the difference between "direct variation" and "inverse variation".
4. Use power functions to solve word problems.

In this section we will study *power functions*.

A power function looks like _____,

where a = _____, and k = _____.

We say that $f(x)$ _____ as the ath power of x, or that $f(x)$ is _____ the ath power of x.

If a is _____ we say that $f(x)$ varies _____ with the ath power of x.

If a is _____ we say that $f(x)$ varies _____ with the ath power of x.

Example 1: Determine if the function is a power function. For those that are not, explain why not.

a) $f(x) = -3x^4$ b) $f(x) = \sqrt[3]{8x^5}$ c) $g(x) = 7 \cdot 2^x$ d) $h(x) = 2x^{-5}$

Example 2: The volume V of a sphere varies directly as the cube of the radius r. When the radius of a sphere is 6 cm, the volume is 904.779 cm^3. What is the radius of a sphere whose volume is 268.083 cm^3?

Example 3: The force of gravity F acting on an object is inversely proportional to the square of the distance d from the object to the center of the earth. Write an equation that models this situation.

Example 4: Velma and Reggie gathered the data in the table below using a 100-watt light bulb and a Calculator-Based Laboratory(CBL) with a light-intensity probe.

a) Use your calculator to find the power regression model of the data.

b) Describe the relationship between the intensity and distance modeled with the equation in part *a*.

c) Use the regression model from part *a* to predict the intensity of an object 2.75 meters away.

Light Intensity Data for a 100-W Light Bulb

Distance(m)	Intensity(W/m^2)
1.0	7.95
1.5	3.53
2.0	2.01
2.5	1.27
3.0	0.90

2.1 LINEAR AND QUADRATIC FUNCTIONS AND MODELING

Learning Targets:

1. Understand what a polynomial function looks like.
2. Understand that Average Rate of Change implies slope between two points.
3. Model Linear functions using function notation and the regression capabilities of your calculator.
4. Find the vertex and graph a quadratic function in standard, intercept, and vertex forms.
5. Model Quadratic functions in vertex form.
6. Use the projectile motion model to find the highest point a projectile reaches, and when it reaches that point.

The rest of Chapter 2 deals with polynomial functions and you will learn about various aspects of these types of functions. Our first goal is to define polynomial functions and then to identify which functions are polynomials.

Definition of a Polynomial Function

A function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$, where n = _____ and n is a _____.

Example 1: Which of the following functions are polynomial functions? For those that are, state the degree and leading coefficient. For those that are not polynomials state why not.

$f(x) = 3x - 5x^2$ $\quad h(x) = \sqrt{x^2 - 4}$ $\quad s(t) = 2t^4 + 5t^3$ $\quad g(x) = 6$ $\quad k(x) = 4x^{\frac{2}{3}} - 5$

For the remainder of this section we will deal only with functions whose degree is less than or equal to 2.

Polynomials of degree 0 are called _____.

Polynomials of degree 1 are called _____.

Polynomials of degree 2 are called _____.

Example 2: The table below shows the relationship between the number of Calories and the number of grams of fat in 9 different hamburgers from various fast-food restaurants.

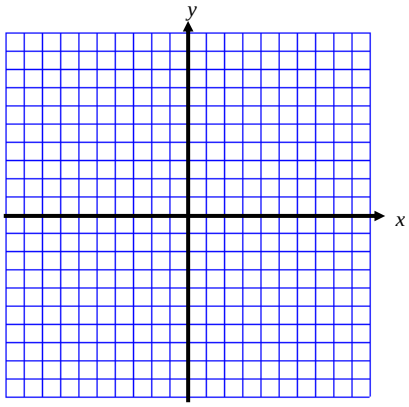
Calories	720	530	510	500	305	410	440	320	598
Fat (g)	46	30	27	26	13	20	25	13	26

a) Find the linear regression model for this data.

b) According to your model, how much fat is in a hamburger with 475 calories? Do NOT round!!

b) In the context of this problem, what does the slope mean?

Example 3: Find the vertex of the quadratic function $f(x) = 3(x+5)^2 - 2$ then graph the quadratic function.



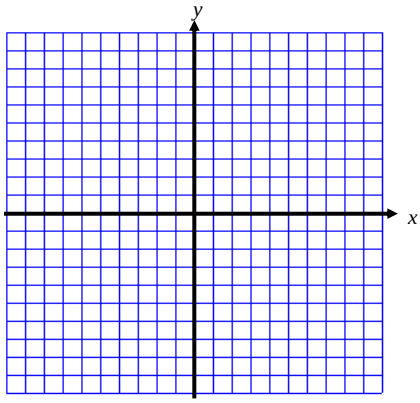
Many of us know to use $x = -\frac{b}{2a}$ to find the vertex when in standard form … $y = ax^2 + bx + c$.
Let's see why this works…
Example 4: Start with the function $f(x) = a(x-h)^2 + k$.

a) Multiply to expand $(x-h)^2$.

b) Distribute the a

c) Compare the coefficients of x^2 and x to a and b from the standard form equation.

Example 5: Find the vertex of the quadratic function $f(x) = 2x^2 - 6x + 11$ and then graph it.



Modeling Quadratic Functions

- In previous courses, we have used quadratic functions to model projectile motion. We will continue to use quadratics in this manner through applications of the vertex and the zeros.
- There are many other situations where we use quadratic functions to model real world applications. One such application is shown below.

Example 6: The Welcome Home apartment rental company has 1600 units available, of which 800 are currently rented at \$300 per month. A market survey indicates that each \$5 decrease in monthly rent will result in 20 new leases.

a) Determine a polynomial function $R(x)$ that models the total rental income realized by Welcome Home, where x is the number of \$5 decreases in monthly rent.

b) Find a graph of $R(x)$ for the rent levels between \$175 and \$300 (that is $0 \le x \le 25$) that clearly shows a maximum for $R(x)$. What rent will yield Welcome Home the maximum monthly income?

2.3 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE WITH MODELING

Learning Targets:

1. Be able to describe the end behavior of any polynomial using limit notation.
2. Be able to find the zeros of a polynomial by factoring.
3. Be able to find the zeros of a polynomial using your graphing calculator.
4. Understand how the multiplicity of a zero changes how the graph behaves when it meets the *x*-axis.
5. Use end behavior and multiplicity of zeros to sketch a polynomial by hand.
6. Use the regression capabilities of your calculator to model a cubic and quartic equation.

Our focus today is on polynomials of degree 3 (cubic), degree 4 (quartic), or higher.

<u>*End behavior*</u> describes what happens to the *y* – values of the function as *x* approaches positive or negative infinity as we learned in lesson 1.2. If you can remember the graphs of $y = x$, $y = -x$, $y = x^2$, and $y = -x^2$, then you can remember the end behavior of ALL polynomial functions using the "leading coefficient" and highest power.

	Even Power	Odd Power
Positive Coefficient	$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$	$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$
Negative Coefficient	$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$	$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$

Example 1: Describe the end behavior of the polynomial function using $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$. Confirm graphically.

a) $f(x) = 3x^6 - 5x + 3$

b) $g(x) = -x^3 + 7x^2 - 8x + 9$

When the end behavior of a function goes to positive or negative infinity, we will write $\lim_{x\to\infty} f(x) = \infty$ or $-\infty$, but since infinity is not a real number, in Calculus we say the limit does not exist.

<u>*Zeros of Polynomial Functions*</u>

The zeros of a function are the *x* values that make f(*x*) = 0. These *x* values are called the *x*-intercepts when they are real numbers. We can use factoring to find these zeros algebraically.

Example 2: Find the zeros of the function algebraically. Support graphically.

a) $f(x) = x^3 - 16x$

Definition: Multiplicity of zeros:

The multiplicity of each zero is the number of times the factor occurs in the factored form of the polynomial.

Example 3: List each zero and state its multiplicity. Then, graph each function on your calculator. What do you notice?

a) $f(x) = x(x-3)^2$

b) $h(x) = (x+1)(x-2)^2(x-4)^3$

- At a zero with EVEN multiplicity, the graph of the function will _____.
- At a zero with ODD multiplicity, the graph of the function will _____.

<u>*Putting it all together*</u>

Example 5: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the *x*-axis at the corresponding *x*-intercept. Using what you know about end behavior and the zeros of the polynomial function, sketch the function.

a) $f(x) = x^4 - 25x^2$

Degree:

Sketch:

Zero(s) & Multiplicity:

End Behavior:

b) $h(x) = -3(x-2)(x+7)^2$

Degree:

Sketch:

Zero(s) & Multiplicity:

End Behavior:

2.4 REAL ZEROS OF POLYNOMIAL FUNCTIONS

Learning Targets:

1. Use long division to find factors of a polynomial.
2. Use synthetic division to find linear factors of a polynomial.
3. Apply the remainder theorem to find the function value at a given value of *x*.
4. Apply the factor theorem to write the factors of a polynomial.

In the last section, we found the zeros of a polynomial by factoring. In this section we explore different ways to obtain those factors. We will focus on long division and synthetic division which should be review from previous courses.

Example 1: Divide.

$$\frac{2x^4 - 3x^3 + 5x - 1}{x - 2}$$

As long as you are dividing by a term that looks like _____, you can also use synthetic division.

Example 2: Divide using synthetic division.

$$\left(2x^4 - 3x^3 + 5x - 1\right) \div \left(x - 2\right)$$

Another way to find the remainder is to evaluate the function using a specific value.

Example 3: Evaluate $f(x) = 2x^4 - 3x^3 + 5x - 1$ when $x = 2$. What do you notice?

The Remainder Theorem

If a polynomial is divided by $(x - c)$, then the remainder is the same as the function value $f(c)$.

When a polynomial is divided by $(x - c)$, we can find the remainder using

a) _____

b) _____

c) _____

Relating Remainders to Factors

- To determine why the remainder is so important, let's review division of numbers.

  When we divide and the remainder is zero, then the value we divided by, as well as the answer, are called factors. For example, when 20 is divided by 5, the remainder is zero. We call 5 a factor of 20. The answer 4 is also a factor of 20.

- The same can be said for polynomials…if a polynomial is divided by $(x - c)$ and the remainder is zero, then $(x - c)$ is a factor of the polynomial and so is the answer obtained from the division.

Relating Factors to Zeros

We also need to understand how the factors of a function are related to its zeros. For example, consider the function $f(x) = x^2 + 7x + 10$. We can easily factor to get $f(x) = (x+5)(x+2)$ and we know the zeros are $x = -5$ and -2.

The Factor Theorem

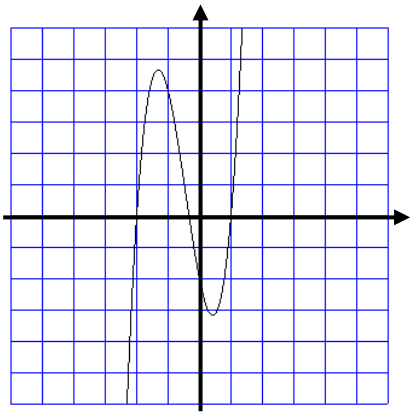
For a polynomial function, $x = c$ is a zero of a function if and only if $(x - c)$ is a factor of the function.

A Quick Summary …The following statements are all equivalent:

1. $x = c$ is a solution (or root) of the equation $f(x) = 0$.
2. When $f(x)$ is divided by $(x - c)$, the remainder equals 0.
3. c is a zero of the function $f(x)$.
4. c is an x-intercept of the graph of $f(x)$ if c is a real number.
5. $(x - c)$ is a factor of $f(x)$.

Example 4: If 5 is a zero of $f(x)$, give four equivalent statements that you know are true.

Example 5: Completely factor $f(x) = 3x^3 + 4x^2 - 5x - 2$. Use the graph below and synthetic division.



Rational Zeros Theorem

Let $f(x)$ be a polynomial function with integral coefficients. The only ***possible rational zeros*** of $f(x)$ are:

$$\frac{p}{q},$$

where p is a divisor of the constant term and q is a divisor of the leading coefficient.

Example 6: Given the function: $f(x) = 6x^3 - 29x^2 - 6x + 5$

a) List all possible rational roots $\frac{p}{q}$ … these are the only *possible* rational zeros of the function.

b) Graph the function to see which **are** the zeros of $f(x)$.
… and realize how nice your teachers are for NOT making you try ALL the numbers in part *c*.

c) Using the graph from part *b* to get you started; find the zeros of $f(x)$ algebraically.
… (use synthetic division and/or factoring)