PreCalculus
Chapter 1 Review Sheet

Name: KEY

No Calculator Allowed

For questions #1 – 3 do the following:
(a) Identify the parent function **(b) Describe the transformation.**
(c) Identify the domain and range and **(d) Accurately graph each function.**

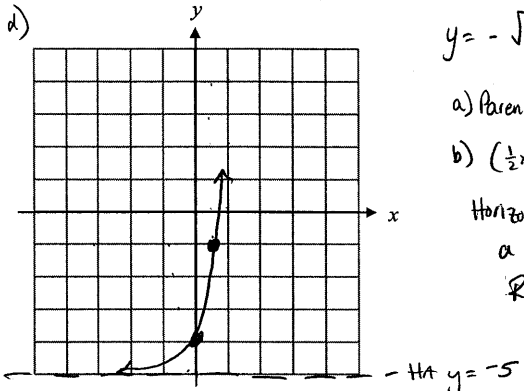
1. $y = 4^{2x} - 5$

a) Parent: $y = 4^x$

b) $(\frac{1}{2}x, y-5)$
Horizontal Shrink by a factor of $\frac{1}{2}$, Down 5

c) D: $\mathbb{R}$ $(-\infty, \infty)$
R: $y > -5$ $(-5, \infty)$

d)



Points HA: $y = 0$
$(0, -4)$
$(\frac{1}{2}, -1)$
Do you have these?

2. $y = -\sqrt{2x-6}$ …** MOST OFTEN MISSED**

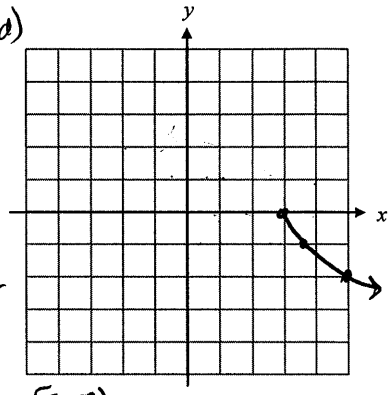
$y = -\sqrt{2(x-3)}$

a) Parent: $y = \sqrt{x}$

b) $(\frac{1}{2}x + 3, -y)$
Horizontal Shrink by a factor of $\frac{1}{2}$, Right 3, Reflect over x-axis

c) D: $x \geq 3$ $[3, \infty)$
R: $y \leq 0$ $(-\infty, 0]$

d)



Do you have these?
Points
$(3, 0)$
$(3\frac{1}{2}, -1)$
$(5, -2)$

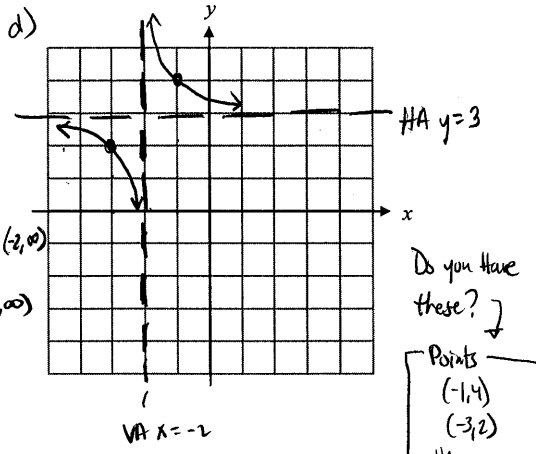
3. $y = \dfrac{1}{x+2} + 3$

a) Parent: $y = \frac{1}{x}$

b) $(x-2, y+3)$
Left 2, Up 3

c) D: $x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$
R: $y \neq 3$ $(-\infty, 3) \cup (3, \infty)$

d)

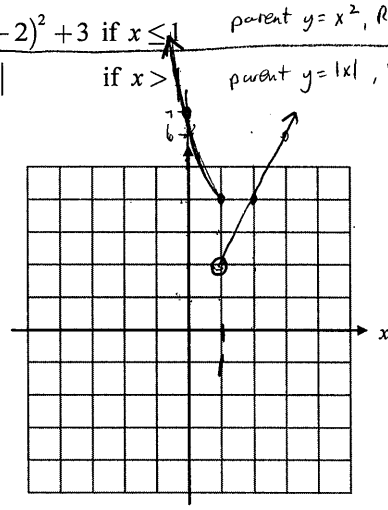


Do you have these?
Points
$(-1, 4)$
$(-3, 2)$
HA: $y = 3$
VA: $x = -2$

4. Sketch: $f(x) = \begin{cases} (x-2)^2 + 3 & \text{if } x \leq 1 \\ 2|x| & \text{if } x > \end{cases}$

parent $y = x^2$, Right 2, Up 3

parent $y = |x|$, vert. stretch by a factor of 2



if $x \leq 1$
Points
$(1, 4)$
$(0, 7)$

if $x > 1$
open $(1, 2)$
pt $(2, 4)$

5. Find the domain of each function.

a) $g(x) = \sqrt{9 - 7x}$

$9 - 7x \geq 0$
$-7x \geq -9$
$x \leq 9/7$

D: $(-\infty, 9/7]$

b) $h(x) = \log_7(2x - 6)$

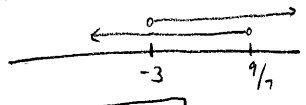
$2x - 6 > 0$
$2x > 6$
$x > 3$

D: $(3, \infty)$

c) $h(x) = \dfrac{\log_7(2x+6)}{\sqrt{9-7x}}$

not equal to 0!

$2x + 6 > 0$ AND $9 - 7x > 0$
$2x > -6$ $-7x > -9$
$x > -3$ $x < 9/7$



D: $(-3, 9/7)$

For questions 6 & 7,

(a) Find the domain

(b) determine if there are any vertical asymptote(s) and/or holes

6. $g(x)=\dfrac{4x^2+6}{x+1}$

a) $x+1\neq 0$

$x\neq -1$

D: $(-\infty,-1)\cup(-1,\infty)$

b) VA at $x=-1$

7. $h(x)=\dfrac{5x^2+2}{x^2-9}=\dfrac{5x^2+2}{(x+3)(x-3)}$

a) $x+3\neq 0$ & $x-3\neq 0$

$x\neq -3$ $\quad x\neq 3$

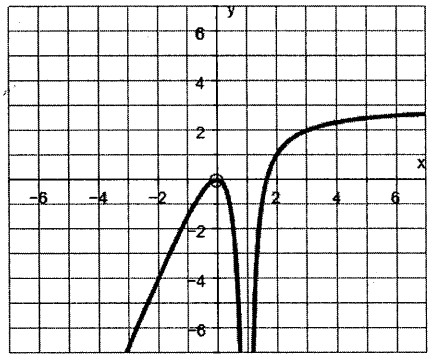
D: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

b) VA at $x=3$ AND $x=-3$

8. Describe the end behavior of the function below using limit notation.

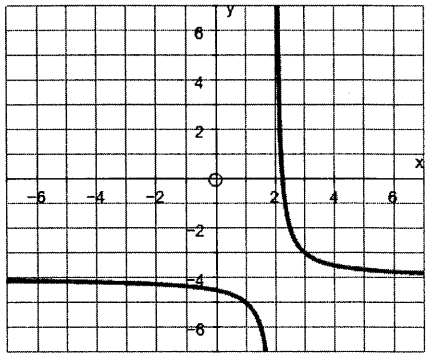
★ see Quiz 1.2 for an example of holes (or WS1.2)

a)



$\lim_{x\to-\infty} f(x) = -\infty$ | $\lim_{x\to\infty} f(x) = 3$

b)

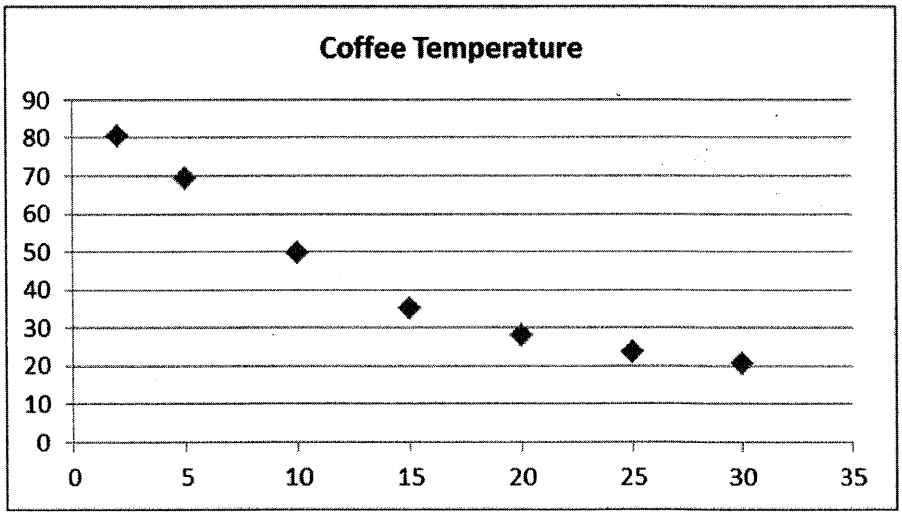


$\lim_{x\to-\infty} f(x) = -4$ | $\lim_{x\to\infty} f(x) = -4$

9. A thermometer is removed from a cup of coffee and place in water with a constant temperature. The temperature was recorded at various times over the next 30 seconds. The results are graphed below.

Which parent function seems to fit the data best? Use vocabulary from this chapter to justify your response.

Looks like an exponential Decay function because the points are decreasing but seem to be reaching a horizontal asymptote (or a lower bound)

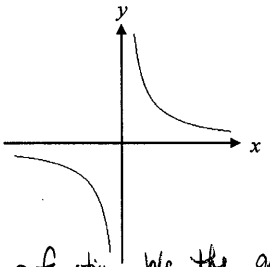


Graphing Calculator Allowed ★

*******KNOW THE 12 PARENT FUNCTIONS ***PROPERTIES***
… BE ABLE TO ANSWER QUESTIONS LIKE THOSE ON YOUR WORKSHEET 1.3.

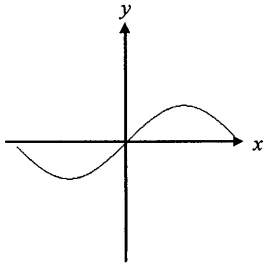
10. Are the given graphs functions? **Explain** why or why not. Do they have an inverse that is also a function. **Explain** why or why not?

a)



- Yes, it is a function b/c the graph passes the VLT
- yes, the inverse is also a function b/c the graph passes the HLT

b)



- Yes the graph is a function, b/c the graph passes the VLT
- No, the inverse of the graph is NOT a function b/c the graph fails the HLT

11. Given $g(x) = 5x + 7$ and $f(x) = 8 - 3x^2$, find $g(f(-2))$.

$f(-2) = 8 - 3(-2)^2$ ⇒ $g(f(-2)) = g(-4) = 5(-4) + 7 = -20 + 7 = \boxed{-13}$

$f(-2) = 8 - 3(4) = -4$

For questions #12 - 19, suppose $f(x) = 2x^2 - 7$ and $g(x) = 3x - 2$. Find each of the following AND state the domain of the new function.

12. $(f+g)(x)$

$f(x) + g(x) = (2x^2 - 7) + (3x - 2)$

$= 2x^2 + 3x - 9$

D: ℝ

13. $(fg)(x)$

$f(x) \cdot g(x) = (2x^2 - 7)(3x - 2)$

$= 6x^3 - 4x^2 - 21x + 14$

D: ℝ

14. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 7}{3x - 2}$

D: $3x - 2 \neq 0$, $3x \neq 2$, $x \neq 2/3$

$(-\infty, 2/3) \cup (2/3, \infty)$

15. $f^{-1}(x)$

$x = 2y^2 - 7$

$x + 7 = 2y^2$

$\frac{x+7}{2} = y^2$

$\sqrt{\frac{x+7}{2}} = y$

$f^{-1}(x) = \sqrt{\frac{x+7}{2}}$

D: $\frac{x+7}{2} \geq 0$, $x + 7 \geq 0$, $x \geq -7$ → $[-7, \infty)$

16. $g(f(x))$

$g(2x^2 - 7) = 3(2x^2 - 7) - 2$

No Domain Restrictions

$= 6x^2 - 21 - 2$

$= 6x^2 - 23$

D: ℝ

17. $\left(\frac{g}{f}\right)(x)$

$\frac{g(x)}{f(x)} = \frac{3x - 2}{2x^2 - 7}$

D: $2x^2 - 7 \neq 0$, $2x^2 \neq 7$, $x^2 \neq 7/2$, $x \neq \pm\sqrt{7/2}$

$(-\infty, -\sqrt{7/2}) \cup (-\sqrt{7/2}, \sqrt{7/2}) \cup (\sqrt{7/2}, \infty)$

18. reflection of $f(x)$ over the x-axis.

$-f(x) = -(2x^2 - 7)$

$= -2x^2 + 7$

19. reflection of $g(x)$ over the y-axis.

$g(-x) = 3(-x) - 2$

$= -3x - 2$

For questions 20 and 21, prove algebraically whether the function is even, odd, or neither.

20. $f(x)=7x^4-x^2$

$f(-x)=7(-x)^4-(-x)^2$

$f(-x)=7x^4-x^2$

$f(-x)=f(x)$

$\therefore f(x)$ is even

21. $f(x)=\frac{3}{4x}$

$f(-x)=\frac{3}{4(-x)}$

$f(-x)=\frac{3}{-4x}=-\frac{3}{4x}$

$f(-x)=-f(x)$

$\therefore f(x)$ is ODD

22. For each of the following,
    i) **Find the inverse** of the function.
    ii) **VERIFY that the function is an inverse** by showing $f(g(x))=x=g(f(x))$

a) $f(x)=3x-2$

i) $x=3y-2$

$x+2=3y$

$\frac{x+2}{3}=y$

$f^{-1}(x)=\frac{x+2}{3}$

ii) $f(f^{-1}(x))=3\left(\frac{x+2}{3}\right)-2$

$=x+2-2$

$=x$

$f^{-1}(f(x))=\frac{(3x-2)+2}{3}$

$=\frac{3x}{3}$

$=x$

b) $h(x)=\frac{2x+4}{5-x}$

i) $x=\frac{2y+4}{5-y}$

$x(5-y)=2y+4$

$5x-xy=2y+4$

$5x-4=2y+xy$

$5x-4=(2+x)y$

$\frac{5x-4}{2+x}=y$

$h^{-1}(x)=\frac{5x-4}{2+x}$

ii) $h(h^{-1}(x))=\frac{2\left(\frac{5x-4}{2+x}\right)+4}{5-\left(\frac{5x-4}{2+x}\right)}=\frac{\frac{2(5x-4)+4(2+x)}{2+x}}{\frac{5(2+x)-(5x-4)}{2+x}}$

$=\frac{10x-8+8+4x}{10+5x-5x+4}$

$=\frac{14x}{4}=x$ ✓

$h^{-1}(h(x))=\frac{5\left(\frac{2x+4}{5-x}\right)-4}{2+\left(\frac{2x+4}{5-x}\right)}=\frac{\frac{5(2x+4)-4(5-x)}{5-x}}{\frac{2(5-x)+(2x+4)}{5-x}}$

$=\frac{10x+20-20+4x}{10-2x+2x+4}$

$=\frac{14x}{14}=x$ ✓

For questions 23 – 25, give the equation of the function whose graph is described.

23. The graph of $f(x)=x^2$ is reflected across the x-axis and vertically stretched by a factor of 7.

$g(x)=-7x^2$

24. The graph of $f(x)=\sqrt{x}$ is shifted 10 units to the left and 7 units upward.

$g(x)=\sqrt{x+10}+7$

25. The graph of $f(x)=|x|$ is horizontally shrunk by a factor of 1/2.

$g(x)=|2x|$

26. Using your calculator, graph the function and
(a) state the intervals on which the function is increasing, decreasing, or constant.
(b) Find all relative maximum and minimum values.

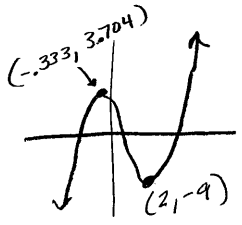
$f(x) = 2x^3 - 5x^2 - 4x + 3$

x-values

INCREASING	DECREASING
$(-\infty, -.333] \cup [2, \infty)$	$[-.333, 2]$

y-values

RELATIVE MAX	RELATIVE MIN
3.704	−9



27. Joe Pearlman received a 3.5% pay raise. His salary after the raise was $37,260. What was his salary before the raise?

x = salary before raise

New = 1.035(old)

$37260 = 1.035x$

$\$36{,}000 = x$

Salary before the raise was $36000.

28. Sue invested $10,000, part at 3.6% annual interest and the balance at 7.8% annual interest. How much invested at each rate if a 1-year interest payment of $667.02.

x = $ invested at 3.6%

y = $ invested at 7.8%

$x + y = 10000$

$.036x + .078y = 667.02$

$y = 10000 - x$

$.036x + .078(10000 - x) = 667.02$

$.036x + 780 - .078x = 667.02$

$-.042x = -112.98$

$x = 2690$

$2690 invested at 3.6%

$7310 invested at 7.8%

29. Thirty kilograms of nickel that was 38% pure were mixed with a nickel alloy that was 28% pure. How many kilograms of each kind were used to produce an alloy that was 32% pure?

x = kg of 28% pure nickel

y = kg of 32% pure nickel

$30 + x = y$

$.38(30) + .28x = .32y$

$.38(30) + .28x = .32(30 + x)$

$11.4 + .28x = 9.6 + .32x$

$1.8 = .04x$

$45 = x$

45 kg of 28% pure nickel were mixed with 30 kg of 38% pure nickel to get 75 kg of an alloy that was 32% pure nickel

CAN YOU?

Can you transform all the parent functions and graph them using correct points?
Can you graph a piecewise function?
Can you find the domain using the 3 domain issues presented in this chapter?
Can you find (and tell the difference) between a vertical asymptote and a hole in a function when given the equation?
Can you describe end behavior of a function using limit notation?
Can you find horizontal asymptotes of rational functions?
Can you use the function notation to add/subtract/multiply/divide functions?
Can you use composite function notation?
Can you prove a function is odd or even?
Can you find an inverse of an equation?
Can you prove an equation is an inverse, and know the difference between proving and finding inverses?
Can you correctly describe when a function is increasing or decreasing?
Can you correctly find and describe the extrema of a function?
Can you explain the difference between absolute and relative (local) extrema?
Can you use the terms, bounded, bounded above, and bounded below correctly?
Can you describe the difference between removable and non-removable discontinuities?
Can you use properties of a graph to identify which parent function would model the graph the best?
Can you do "mixture" problems?

Pre-Requisite Review (Yeah … anything from chapter P could be on the chapter 1 exam too!)REVIEW YOUR WORKSHEETS, QUIZZES, AND NOTES … and don't forget about that 1.3 WS specifically!