

# PreCalculus Second Semester Review

## Ch. P to Ch. 3 (1st Semester) ~ No Calculator

KEY

Solve. Express answer using interval notation where appropriate. Check for extraneous solutions.

P3 1.  $\left(\frac{x-2}{3} - \frac{x+5}{2} = \frac{1}{3}\right)$  6

$$2(x-2) - 3(x+5) = 2$$

$$2x - 4 - 3x - 15 = 2$$

$$-x - 19 = 2$$

$$-x = 21$$

$x = -21$

P5 2.  $\frac{3x}{x+1} + \frac{5}{x-2} = \frac{15}{x^2-x-2}$

$$\left(\frac{3x}{x+1} + \frac{5}{x-2} = \frac{15}{(x-2)(x+1)}\right) (x+1)(x-2)$$

$$3x(x-2) + 5(x+1) = 15$$

$$3x^2 - 6x + 5x + 5 = 15$$

$$3x^2 - x - 10 = 0$$

$$(3x+5)(x-2) = 0$$

$x = -\frac{5}{3}$

 or  ~~$x = 2$~~  extraneous

2.4 3. Find the zeros of the function  
 $f(x) = x^4 + 3x^3 - 3x^2 + 3x - 4$   
 given zeros  $x = 1$  and  $x = -4$

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -3 & 3 & -4 \\ & & 1 & 4 & 1 & 4 \\ \hline -4 & 1 & 4 & 1 & 4 & 0 \\ & & -4 & 0 & -4 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

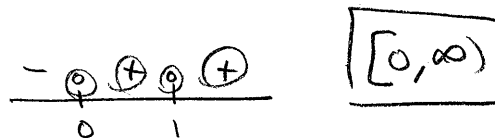
$$x = \pm\sqrt{-1} = \pm i$$

2.8 4.  $x^3 - 2x^2 + x \geq 0$

$$x(x^2 - 2x + 1) \geq 0$$

$$x(x-1)(x-1) \geq 0$$

USE SIGN CHART!



P1 Simplify. Express your answer without negative exponents.

5.  $\frac{(uv^{-2})^{-3}}{u^{-5}v^2} = \frac{u^{-3}v^6}{u^{-5}v^2} = \frac{u^2v^4}{u^3v^2} = \boxed{u^2v^4}$

6.  $\frac{4a^3b}{a^2b^3} \cdot \frac{3b^2}{2a^2b^4} = \frac{12a^3b^3}{2a^4b^7} = \boxed{\frac{6}{ab^4}}$

P4 7. Write the equation of a line a) parallel to and b) perpendicular to  $5x - y = 7$  and passing through the point  $(3, -4)$ .

↓  
slope = 5

↓  
slope =  $-\frac{1}{5}$

$$-y = 7 - 5x$$

$$y = -7 + 5x$$

Slope = 5

a)  $y + 4 = 5(x - 3)$

b)  $y + 4 = -\frac{1}{5}(x - 3)$

1.2 Prove algebraically whether the function is even, odd, or neither.

8.  $f(x) = 3x^3 - 2x$

$$f(-x) = 3(-x)^3 - 2(-x)$$

$$f(-x) = 3(-x^3) + 2x$$

$$f(-x) = -3x^3 + 2x$$

$$f(-x) = -f(x) \therefore \underline{f(x) \text{ is odd}}$$

9.  $f(x) = -2x^4 - 4x + 7$

$$f(-x) = -2(-x)^4 - 4(-x) + 7$$

$$f(-x) = -2(x^4) + 4x + 7$$

$$f(-x) = -2x^4 + 4x + 7$$

$$f(-x) \neq f(x) \text{ and } f(-x) \neq -f(x) \therefore \underline{f(x) \text{ is NEITHER}}$$

1.2 Find the domain. Express the answer in interval notation.

10.  $g(x) = \sqrt{6-5x}$  → CANNOT BE NEGATIVE!

$$\begin{aligned} 6-5x &\geq 0 \\ -5x &\geq -6 \\ x &\leq 6/5 \end{aligned}$$

Domain  
 $\boxed{(-\infty, \frac{6}{5}]}$

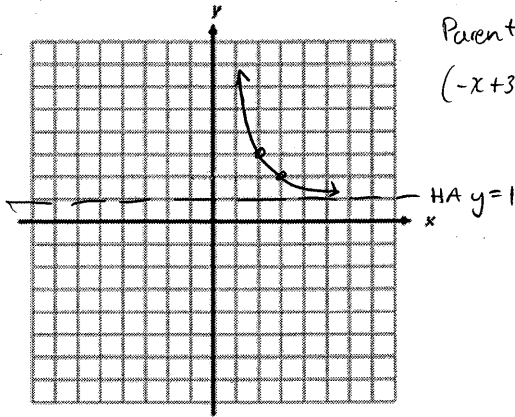
11.  $f(x) = \log(x+3)$  cannot be zero & cannot be negative

$$\begin{aligned} x+3 &> 0 \\ x &> -3 \end{aligned}$$

Domain  
 $\boxed{(-3, \infty)}$

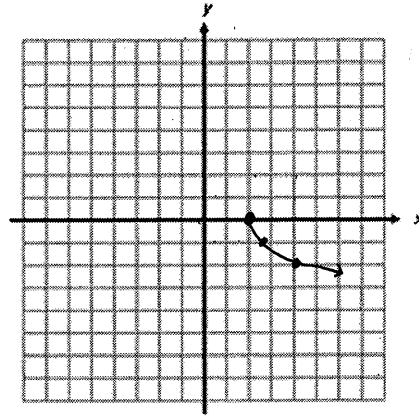
1.5 Identify the parent function, as well as the transformation. Then, graph the function including key points and/or asymptotes.

12.  $f(x) = 2^{3-x} + 1 = 2^{-x+3} + 1 = 2^{-(x-3)} + 1$



Parent:  $y = 2^x$   
 $(-x+3, y+1)$

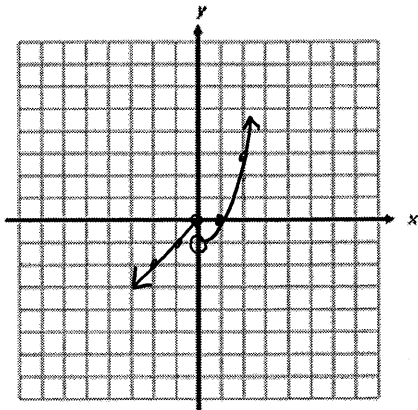
13.  $g(x) = -\sqrt{2x-4} = -\sqrt{2(x-2)}$



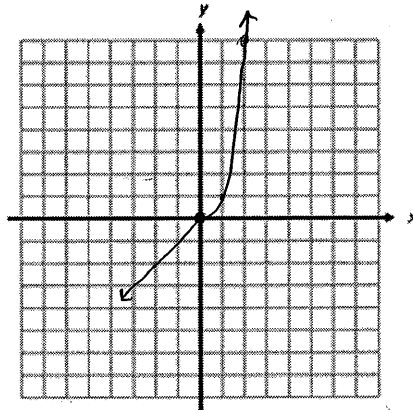
$(\frac{1}{2}x + 2, -y)$   
 Parent:  $y = \sqrt{x}$

1.3 Graph the piecewise-defined function. State whether the function is continuous or discontinuous at  $x = 0$ .

14.  $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 - 1 & \text{if } x > 0 \end{cases}$



15.  $f(x) = \begin{cases} -|x| & \text{if } x < 0 \\ x^3 & \text{if } x \geq 0 \end{cases}$



1.4 16. Find the inverse of  $h(x) = \sqrt[3]{6x-1}$

$$\begin{aligned} x &= \sqrt[3]{6y-1} \\ x^3 &= 6y-1 \\ x^3+1 &= 6y \\ \frac{x^3+1}{6} &= y \\ \therefore h^{-1}(x) &= \frac{x^3+1}{6} \end{aligned}$$

2.5 17. Simplify. Express the answer in a + bi form.

$$\frac{(2+3i)(1+2i)}{(1-2i)(1+2i)} = \frac{2+4i+3i+6i^2}{1-4i^2} = \frac{2+7i-6}{1+4} = \frac{-4+7i}{5} = \boxed{-\frac{4}{5} + \frac{7}{5}i}$$

2.7 Find (if it exists) the a) asymptotes, b) intercepts, and c) domain of the function. Be sure to list any holes.

$$18. g(x) = \frac{x^2-9}{2x^2-x-15} = \frac{(x+3)(x-3)}{(2x+5)(x-3)}$$

Domain:  $x \neq -5/2$  &  $x \neq 3$   
 $(-\infty, -5/2) \cup (-5/2, 3) \cup (3, \infty)$

VA:  $x = -5/2$

HA:  $y = 1/2$

Hole when  $x=3$  @  $(3, 3/11)$

x-int:  $(-3, 0)$

y-int:  $(0, 3/5)$

3.3 19. Simplify each expression.

(a)  $\log_5 1 = \boxed{0}$

(b)  $\log \sqrt[4]{10}$

$\log 10^{1/4} = \boxed{1/4}$

(c)  $3^{\log_3 7} = \boxed{7}$

3.4 20. Expand each logarithm:

(a)  $\log_2 \left( \frac{8\sqrt[5]{x}}{y} \right)$

$\log_2 8 + \log_2 x^{1/5} - \log_2 y$

$\boxed{3 + \frac{1}{5} \log_2 x - \log_2 y}$

(b)  $\log \left( \frac{\sqrt{x^5}}{10} \right)$

$\log x^{5/2} - \log 10$

$\boxed{\frac{5}{2} \log x - 1}$

(c)  $\ln(6x^4e^3)$

$\ln 6 + \ln x^4 + \ln e^3$

$\boxed{\ln 6 + 4 \ln x + 3}$

Calculator Allowed.

3.2 21. A radioactive isotope decays at a rate of 3% per day. A scientist has an initial amount of 50 g. Write a model for this situation. Determine approximately how many days it will take for half the isotope to decay.

$y = 50(.97)^x$

$x = \# \text{ of days}$

$y = \text{amount left after } x \text{ days}$

$25 = 50(.97)^x$

$\frac{1}{2} = (.97)^x$

$\log_{.97} \left( \frac{1}{2} \right) = x$

$\boxed{x \approx 22.757 \text{ days}} \text{ until } 1/2 \text{ has decayed}$

3.5 22. Solve algebraically:

(a)  $\log_3 x + \log_3(x+8) = 2$

$\log_3(x^2+8x) = 2$

$q = x^2+8x$   
 $0 = x^2+8x-9$   
 $0 = (x+9)(x-1)$

(c)  $3^{\frac{x}{2}} - 6 = 42$

$x = -9$  or  $x = 1$   
 extraneous

$3^{x/2} = 48$

$\frac{x}{2} = \log_3(48)$

$x = 2 \log_3(48)$

$x \approx 7.047438029$

(b)  $\log_2(x+5) - \log_2 x = 7$

$\log_2\left(\frac{x+5}{x}\right) = 7$

$2^7 = \frac{x+5}{x}$

$128x = x+5$   
 $127x = 5$

$x = 5/127$

(d)  $-27 = -3 \cdot \left(\frac{1}{4}\right)^{6x}$

$9 = \left(\frac{1}{4}\right)^{6x}$

$\log_{\frac{1}{4}}(9) = 6x$

$\frac{\log_{1/4}(9)}{6} = x$

$x \approx -0.2641604168$

Ch. 4, Ch. 5, 9.2, 9.4, 6.1 and 6.3 (2<sup>nd</sup> Semester) ~ No Calculator

4.2 23. Find each exact value.

4.3

(a)  $\cos\left(\frac{3\pi}{4}\right)$

(b)  $\sin\left(-\frac{7\pi}{6}\right)$

(c)  $\tan\left(\frac{3\pi}{2}\right)$

(d)  $\cos\left(\frac{-7\pi}{3}\right)$

$\frac{-1}{\sqrt{2}}$  or  $-\frac{\sqrt{2}}{2}$

$\frac{1}{2}$

$\frac{-1}{0}$  → undefined

$\frac{1}{2}$

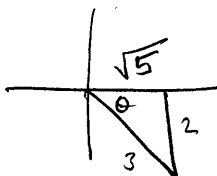
4.3 24. Find one positive angle and one negative angle that are coterminal with:  $\frac{3\pi}{4}$

$\frac{11\pi}{4}$

$-\frac{5\pi}{4}$

4.3 25. Given:  $\sin \theta = -\frac{2}{3}$  and  $\cos \theta > 0$  Find the values of the remaining five trigonometric functions of  $\theta$ .

QIV

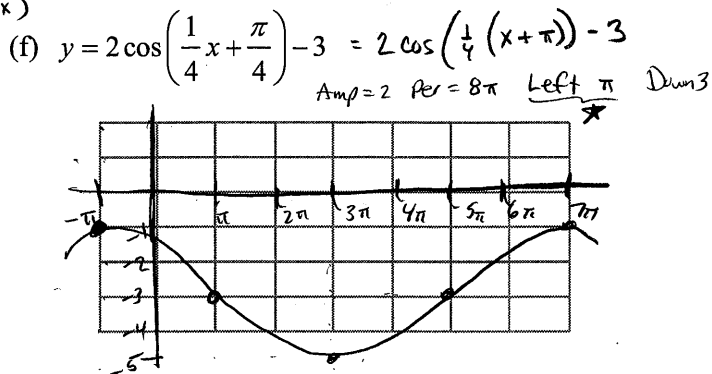
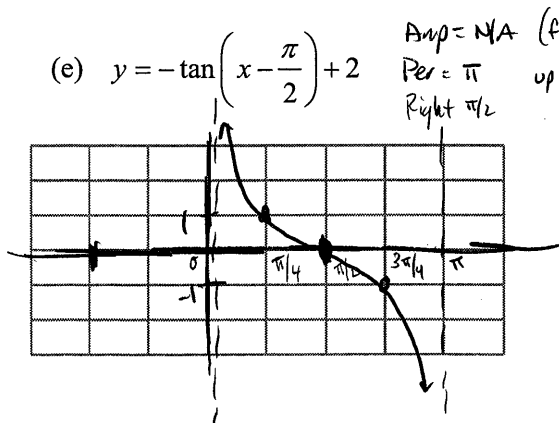
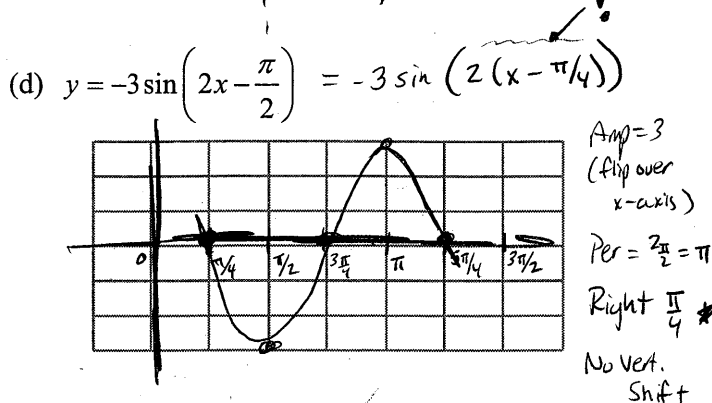
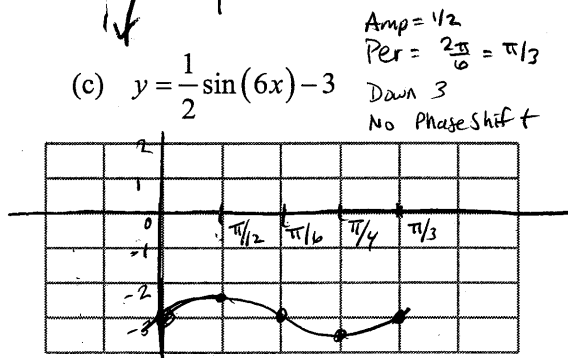
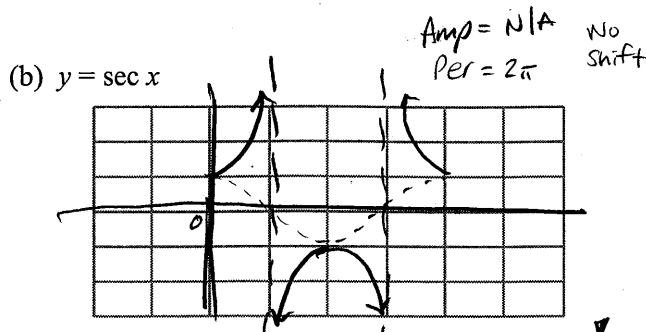
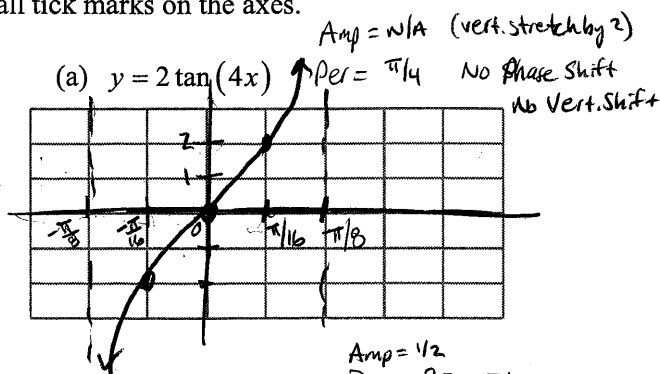


$\sin \theta = -\frac{2}{3}$      $\csc \theta = -\frac{3}{2}$

$\cos \theta = \frac{\sqrt{5}}{3}$      $\sec \theta = \frac{3}{\sqrt{5}}$

$\tan \theta = -\frac{2}{\sqrt{5}}$      $\cot \theta = -\frac{\sqrt{5}}{2}$

4.4 26. State the amplitude, period, phase shift, and vertical shift, and then graph each function. Clearly label all tick marks on the axes.



4.4 27. Write an equation of the cosine function with amplitude = 2, period =  $\frac{\pi}{2}$ , phase shift =  $-\frac{\pi}{8}$  and vertical shift = -3.

$$y = 2 \cos\left(4\left(x + \frac{\pi}{8}\right)\right) - 3$$

4.4 28. Use the given graph...

5.3 (a) Write a sine function that fits the graph. Left  $2.5\pi$

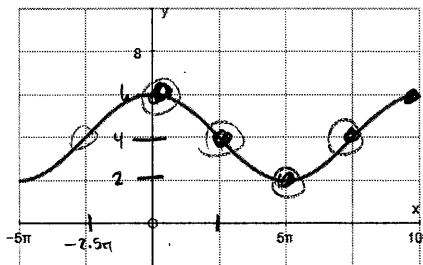
$$y = 2 \sin\left(\frac{1}{5}\left(x + \frac{5\pi}{2}\right)\right) + 4$$

(b) Write a cosine function that fits the graph.

$$y = 2 \cos\left(\frac{1}{5}x\right) + 4$$

(c) Using identities, prove that the two equations you wrote are equal.

$$\begin{aligned} 2 \sin\left(\frac{1}{5}\left(x + \frac{5\pi}{2}\right)\right) + 4 &= 2 \sin\left(\frac{1}{5}x + \frac{\pi}{2}\right) + 4 \\ &= 2 \left[ \sin\left(\frac{1}{5}x\right) \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{1}{5}x\right) \sin\left(\frac{\pi}{2}\right) \right] + 4 \\ &= 2 \left[ \sin\left(\frac{1}{5}x\right) \cdot 0 + \cos\left(\frac{1}{5}x\right) \cdot 1 \right] + 4 \\ &= 2 \cos\left(\frac{1}{5}x\right) + 4 \quad \checkmark \end{aligned}$$



4.7 29. Find each value.

(a)  $\arccos\left(\frac{1}{\sqrt{2}}\right)$

$\pi/4$

(b)  $\cos^{-1}\left(-\frac{1}{2}\right)$

$2\pi/3$

(c)  $\sec\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$\sec\left(\pi/3\right) = \frac{1}{\cos(\pi/3)}$

$2$

(d)  $\sin\left[\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$

$\sin\left(-\frac{\pi}{6}\right)$

$-\frac{1}{2}$

(e)  $\cos[\arcsin(-1)]$

$\cos\left(\frac{\pi}{2}\right)$

$0$

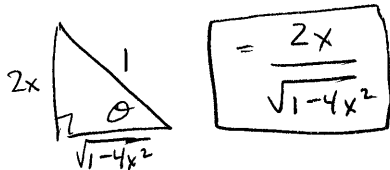
(f)  $\sin^{-1}\left[\sin\left(\frac{5\pi}{6}\right)\right]$

$\sin^{-1}\left(\frac{1}{2}\right)$

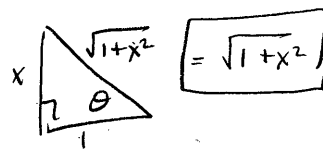
$\pi/6$

4.7 30. Write an algebraic expression equivalent to each expression below. Hint: Draw a  $\Delta$ .

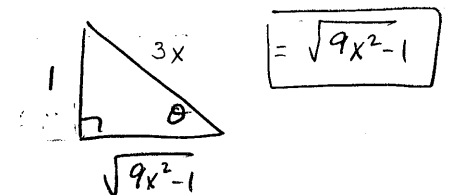
(a)  $\tan(\sin^{-1} 2x) = \tan(\theta)$



(b)  $\sec(\tan^{-1} x) = \sec(\theta)$



(c)  $\cot(\csc^{-1} 3x) = \cot(\theta)$



5.1-5.4 Verify that each of the following is an identity. Be sure to show all steps.

31.  $\sin^2 \theta (\csc^2 \theta - 1) + \tan(-\theta) \cos \theta + \cos\left(\frac{\pi}{2} - \theta\right) = \cos^2 \theta$   
 $= \sin^2 \theta (\cot^2 \theta) - \tan \theta \cos \theta + \sin \theta$   
 $= \cancel{\sin^2 \theta} \cdot \frac{\cos^2 \theta}{\cancel{\sin^2 \theta}} - \frac{\cancel{\sin \theta}}{\cancel{\cos \theta}} \cdot \cancel{\cos \theta} + \cancel{\sin \theta}$   
 $= \cos^2 \theta - \cancel{\sin \theta} + \cancel{\sin \theta}$   
 $= \boxed{\cos^2 \theta}$

32.  $\frac{\sin \beta}{\csc \beta} + \frac{\cos \beta}{\sec \beta} = 1$   
 $= \frac{\sin \beta}{\frac{1}{\sin \beta}} + \frac{\cos \beta}{\frac{1}{\cos \beta}}$   
 $= \sin^2 \beta + \cos^2 \beta$   
 $= 1$

33.  $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

$\frac{2 \tan x}{\sec^2 x}$

$2 \tan x \div \sec^2 x$

$\frac{2 \sin x}{\cos x} \div \frac{1}{\cos^2 x}$

$\frac{2 \sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos^2 x}}{1} = 2 \sin x \cos x = \sin(2x)$

34.  $\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2 \sec x$

$= \frac{\cos x(1 - \sin x) + \cos x(1 + \sin x)}{(1 + \sin x)(1 - \sin x)}$

$= \frac{\cancel{\cos x} - \cancel{\cos x} \sin x + \cancel{\cos x} + \cancel{\cos x} \sin x}{1 - \sin^2 x}$

$= \frac{2 \cos x}{\cos^2 x}$

$= \frac{2}{\cos x} = 2 \sec x$

35.  $\sin(\pi - x) = \sin x$

$= \sin(\pi) \cos(x) - \cos(\pi) \sin x$   
 $= 0 \cdot \cos(x) - (-1) \sin x$   
 $= \sin x$

5.3 36. Find the exact value of  $\cos 105^\circ$ .

$$\begin{aligned}\cos(45+60) &= \cos(45)\cos(60) - \sin(45)\sin(60) \\ &= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}\end{aligned}$$

5.3- Rewrite using identities, then simplify if possible.

5.4

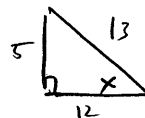
37.  $1 - 2\sin^2 150^\circ$

$$\begin{aligned}&= \cos(2 \cdot 150) \\ &= \boxed{\cos(300^\circ)} = \boxed{\frac{1}{2}}\end{aligned}$$

38.  $\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{5}\right)\sin\left(\frac{\pi}{2}\right)$

$$\sin\left(\frac{\pi}{5} + \frac{\pi}{2}\right) = \boxed{\sin\left(\frac{7\pi}{10}\right)}$$

5.4 39. If  $\cos x = -\frac{12}{13}$  and  $x$  is in the second quadrant, find



(a)  $\sin(2x)$

$$\begin{aligned}&2\sin x \cos x \\ &2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right) \\ &= \boxed{-\frac{120}{169}}\end{aligned}$$

$$\begin{aligned}\sin x &= \frac{+5}{13} \\ \tan x &= -\frac{5}{12}\end{aligned}$$

(b)  $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

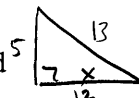
$$= \frac{2(-5/12)}{1 - (-5/12)^2} = \frac{-5/6}{1 - 25/144}$$

$$= -\frac{5}{6} \cdot \frac{144}{144} = -\frac{5 \cdot 24}{144}$$

$$= -\frac{5 \cdot 24}{144} = -\frac{120}{144}$$

$$= \boxed{-\frac{120}{119}}$$

5.4 40. If  $\cot x = -\frac{12}{5}$  and  $x$  is in the fourth quadrant, find



(a)  $\sin(2x)$

$$\begin{aligned}&2\sin x \cos x \\ &2\left(-\frac{5}{13}\right)\left(\frac{12}{13}\right) \\ &= \boxed{-\frac{120}{169}}\end{aligned}$$

$$\begin{aligned}\cos x &= \frac{12}{13} \\ \sin x &= -\frac{5}{13}\end{aligned}$$

(b)  $\cos(2x)$

$$\begin{aligned}&\cos^2 x - \sin^2 x \\ &\left(\frac{12}{13}\right)^2 - \left(-\frac{5}{13}\right)^2 \\ &= \frac{144 - 25}{169} = \boxed{\frac{119}{169}}\end{aligned}$$

Ch5 41. Solve each equation for  $[0, 2\pi)$ .

(a)  $2\sin^2 x = \sqrt{3}\sin x$

$$2\sin^2 x - \sqrt{3}\sin x = 0$$

$$\sin x (2\sin x - \sqrt{3}) = 0$$

$$\sin x = 0 \quad \text{or} \quad \sin x = \frac{\sqrt{3}}{2}$$

$$\boxed{x = 0, \pi \quad \text{or} \quad x = \frac{\pi}{3}, \frac{2\pi}{3}}$$

(b)  $8\cos^2 x = 4$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

(c)  $\cos(2x) + \sin(x) = 0$

$$1 - 2\sin^2 x + \sin x = 0$$

$$0 = 2\sin^2 x - \sin x - 1$$

$$0 = (2\sin x + 1)(\sin x - 1)$$

$$\sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = 1$$

$$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \text{or} \quad x = \frac{\pi}{2}}$$

(d)  $\sin(2x) - 2\cos(x) = 0$

$$2\sin x \cos x - 2\cos x = 0$$

$$2\cos x (\sin x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad \sin x = 1$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad x = \frac{\pi}{2}}$$

Ch5 42. Solve for all values of  $x$ .

(a)  $\cos^2 x - 2\sin^2 x + 2 = 0$

$(1 - \sin^2 x) - 2\sin^2 x + 2 = 0$

$3 - 3\sin^2 x = 0$

$3 = 3\sin^2 x$

$1 = \sin^2 x$

$\pm 1 = \sin x$

$x = \pi/2 + 2\pi \cdot n \quad n = \text{integer}$   
 $x = 3\pi/2 + 2\pi \cdot n$

Calculator Allowed

(b)  $\sin\left(\frac{3\pi}{2} - x\right) = 1$

$\sin\left(\frac{3\pi}{2}\right)\cos x - \cos\left(\frac{3\pi}{2}\right)\sin x = 1$

$-1 \cdot \cos x - 0 \cdot \sin x = 1$

$\cos x = -1$

$x = \pi + 2\pi \cdot n$

4.1 43. The wheel (including the tire) of a sports car under development by an auto company has an eleven inch radius. How many rpm's does the wheel make at 55 mph? rev/min

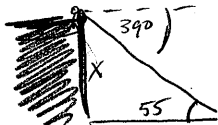
$\frac{55 \text{ mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ rev}}{22\pi \text{ in}} \approx$

4.1 44. Find the measure of the intercepted arc in terms of  $\pi$  in a circle with diameter 60 inches and central angle of 72°.  $r = 30$

$72 \cdot \frac{\pi}{180} = \frac{2\pi}{5}$

$s = \frac{2\pi}{5} \cdot 30 = 12\pi$

4.6 45. The angle of depression from the top of a building to a point 55 feet away from the building (on level ground) is  $39^\circ$ . Determine the height of the building.

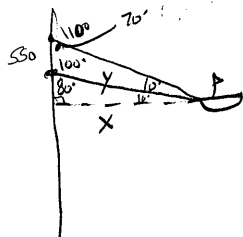


$\tan 39^\circ = \frac{x}{55}$

$55 \tan 39^\circ = x$

$x \approx 44.538 \text{ feet}$

4.6 46. A shoreline runs north-south, and a boat is due east of the shoreline. The bearings of the boat from two points on the shore are  $110^\circ$  and  $100^\circ$ . Assume the two points are 550 feet apart. How far is the boat from the shore?



$\frac{\sin 10^\circ}{550} = \frac{\sin 70^\circ}{y}$

$y \approx 2976.31077$  } store as A

$\sin 80^\circ = \frac{x}{y} \Rightarrow x = y \cdot \sin 80^\circ \approx 2931.094 \text{ feet}$

5.6 47. Find the area of each triangle.

(a)  $a = 7, b = 12, c = 13$

$S = \frac{7 + 12 + 13}{2} = 16$

$A = \sqrt{16(16-7)(16-12)(16-13)}$

$A \approx 41.569 \text{ units}^2$

(b)  $A = 47^\circ, b = 32, c = 19$

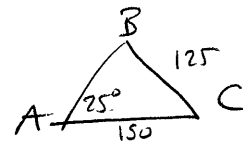
$A = \frac{1}{2}(32)(19) \sin(47^\circ)$

$A \approx 222.332 \text{ units}^2$



5.5 48. Solve each triangle. If there are two  $\Delta$ 's, solve both!

5.6

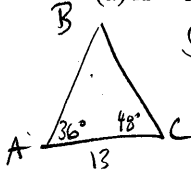


"ASS"

D, 1, or 2  $\Delta$ 's?

(a)  $A = 36^\circ, b = 13, C = 48^\circ$

(b)  $a = 125, A = 25^\circ, b = 150$



(ASA) - 1  $\Delta$   $\boxed{B = 96^\circ}$

$$\frac{\sin 96}{13} = \frac{\sin 48}{c}$$

$\boxed{c \approx 9.714}$

$$\frac{\sin 36}{a} = \frac{\sin 96}{13}$$

$\boxed{a \approx 7.683}$

(c)  $a = 1.5, b = 2.3, c = 1.9$

SSS

$$2.3^2 = 1.5^2 + 1.9^2 - 2(1.5)(1.9)\cos B$$

$$\cos^{-1}\left(\frac{2.3^2 - 1.5^2 - 1.9^2}{-2(1.5)(1.9)}\right) = B$$

$\boxed{84.261^\circ \approx B}$

$$1.9^2 = 1.5^2 + 2.3^2 - 2(1.5)(2.3)\cos C$$

$$\cos^{-1}\left(\frac{1.9^2 - 1.5^2 - 2.3^2}{-2(1.5)(2.3)}\right) = C$$

$\boxed{C \approx 55.280^\circ}$

$\boxed{A \approx 40.459^\circ}$

$$\frac{\sin 25^\circ}{125} = \frac{\sin B}{150}$$

$B \approx 30.4736409^\circ$   
 $C \approx 124.5263591^\circ$

or  $B \approx 149.5263591^\circ$   
or  $C \approx 5.473640898^\circ$

$$\frac{\sin C}{c} = \frac{\sin 25^\circ}{125}$$

1st A

$$\frac{\sin C}{c} = \frac{\sin 25^\circ}{125}$$

2nd A

$\boxed{c \approx 243.6789885}$

or  $\boxed{c \approx 28.21334757}$

9.4 49. The sequence  $\{2, 6, 18, 54, \dots\}$  is geometric. Find

(a) a recursive rule for the nth term.

$a_1 = 2$   
 $a_n = a_{n-1} \cdot 3 \quad n \geq 2$

(b) an explicit formula for the nth term.

$a_n = 2 \cdot 3^{n-1}$

9.4 50. Suppose an arithmetic sequence contains  $a_{18} = 49$  and  $a_{52} = 174.8$ . Find ...

(a) the common difference  $\boxed{3.7}$

$$49 + d(34) = 174.8$$

$$a_1 + 3.7(17) = 49$$

(b)  $a_1$   $\boxed{-13.9}$

$$34d = 125.8$$

$$a_1 + 62.9 = 49$$

$$d = 3.7$$

$$a_1 = -13.9$$

(c) a recursive formula

$a_1 = -13.9$   
 $a_n = a_{n-1} + 3.7 \quad n \geq 2$

9.4 51. The fourth and ninth terms of a geometric sequence are 128 and 131072 respectively. Find

a) the common ratio  $\boxed{4}$

$$128 \cdot r^5 = 131072$$

$$a_1 \cdot r^3 = 128$$

b)  $a_1$   $\boxed{2}$

$$r^5 = 1024$$

$$a_1 \cdot 4^3 = 128$$

c) an explicit formula

$$r = 4$$

$$a_1 = 2$$

$a_n = 2 \cdot 4^{n-1}$

9.4 52. Write each series using sigma notation.

(a)  $2 + 5 + 8 + 11 + 14 + \dots$

$a_1 = 2$   
 $d = 3$

$$\sum_{n=1}^{\infty} 2 + 3(n-1)$$

(b)  $6 + 2 + \frac{2}{3} + \frac{2}{9} + \dots + \frac{2}{729}$

$a_1 = 6$   
 $r = \frac{1}{3}$

$$\sum_{n=1}^8 6\left(\frac{1}{3}\right)^{n-1}$$

$6\left(\frac{1}{3}\right)^{n-1} = \frac{2}{729}$

$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$

$\log_{\frac{1}{3}}\left(\frac{1}{2187}\right) = n-1$

$7 = n-1$

$8 = n$

9.4 53. Find the sum of the terms of the arithmetic sequence:  $\{28, 22, 16, 10, \dots, -38\}$

$? = \frac{n(28 + -38)}{2}$

$Sum = \frac{12(28 + -38)}{2} = \boxed{-60}$

$-38 = 28 - 6(n-1)$

$-66 = -6(n-1)$

$+11 = n-1$

$12 = n$

For #54 and #55, tell whether the series converges or diverges. If the series converges, find its sum. If the series diverges, explain why.

9.4 54.  $\frac{1}{7} + \frac{5}{14} + \frac{25}{28} + \dots$

$|r| = \left|\frac{5}{2}\right| > 1$   
Divergent

55.  $\frac{1}{3} - \frac{2}{9} + \frac{4}{27} - \dots$

$|r| = \left|-\frac{2}{3}\right| < 1$  Convergent

$Sum = \frac{\frac{1}{3}}{1 - (-\frac{2}{3})} = \frac{\frac{1}{3}}{\frac{5}{3}} = \boxed{\frac{1}{5}}$

9.2 56. Expand:

(a)  $(x-2)^4$

$1x^4(-2)^0 + 4x^3(-2)^1 + 6x^2(-2)^2 + 4x^1(-2)^3 + 1x^0(-2)^4$

$x^4 - 8x^3 + 24x^2 - 32x + 16$

(b)  $(2x+y)^5$

$1(2x)^5(y)^0$   
 $5(2x)^4(y)^1$   
 $10(2x)^3(y)^2$   
 $10(2x)^2(y)^3$   
 $5(2x)^1(y)^4$   
 $1(2x)^0(y)^5$

$32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$

9.2 57. Find the coefficient of the  $x^8y^3$  term in  $(x+y)^{11}$

${}^{11}C_3 x^8 y^3$      $165$

6.1 58. A vector  $u$  has a magnitude of 3.4 cm and a direction of  $27^\circ$ . Write the component form for this vector.

$\langle 3.4 \cos 27^\circ, 3.4 \sin 27^\circ \rangle$

6.1 59. Given points  $A(-6, 2)$  and  $B(1, -3)$ . Find the component form and magnitude of  $\vec{AB}$ .

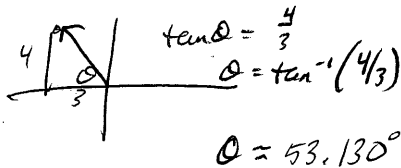
$+6 - 2$        $+1 - 2$

$\langle 7, -5 \rangle$

$|\vec{AB}| = \sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74}$

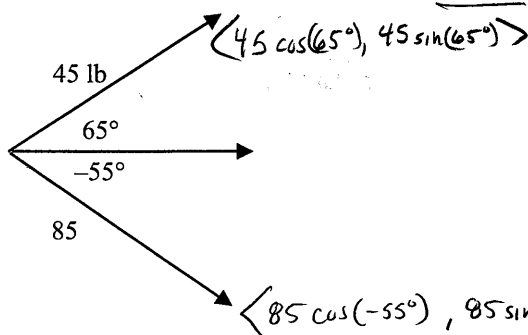
6.1 60. Given  $v = \langle -3, 4 \rangle$ . Find  $|v|$  and give the direction angle in  $[0, 360^\circ)$ .

$|v| = \sqrt{9 + 16} = 5$



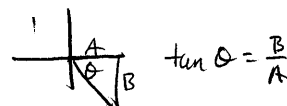
Direction =  $126.870^\circ$

6.1 61. A force of 45 lb acts on an object at an angle of  $65^\circ$ . A second force of 85 lb acts on the object at an angle of  $-55^\circ$ . Find the direction and magnitude of the resultant force.



$\langle 45 \cos(65^\circ) + 85 \cos(-55^\circ), 45 \sin(65^\circ) + 85 \sin(-55^\circ) \rangle$   
 $A \approx 67.771, \dots$        $B \approx -28.844, \dots$

magnitude =  $\sqrt{A^2 + B^2} = 73.655$



Direction  $\theta = \tan^{-1}(B/A) \approx -23.055^\circ$

6.3 62. Eliminate the parameter and identify the graph of the curve.

a)  $x = t - 4, y = t^2$

$x + 4 = t$   
 $y = (x + 4)^2$   
Parabola

b)  $x = 4 \sin t, y = 4 \cos t$

$\frac{x}{4} = \sin t$        $\frac{y}{4} = \cos t$

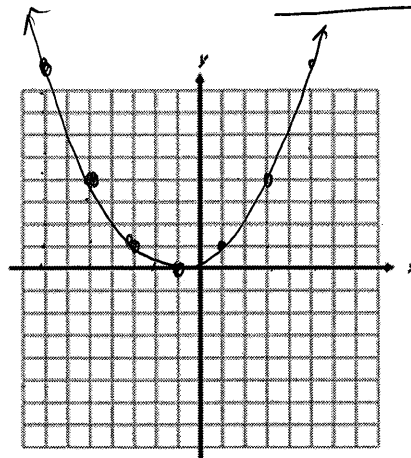
$\sin^2 t + \cos^2 t = 1$

$\frac{x^2}{16} + \frac{y^2}{16} = 1$

$x^2 + y^2 = 16$   
Circle  $C(0,0)$   
 $r = 4$

6.3 63. Create a table of values and the graph of the pair of parametric equations  $x = 2t - 1, y = t^2$

t	x	y
-3	-7	9
-2	-5	4
-1	-3	1
0	-1	0
1	1	1
2	3	4
3	5	9



6.3

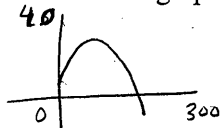
64. Gretchen Austgen, an outfielder for the West Chicago Wildcats, is 215 feet from home plate after catching a fly ball. The runner tags third and heads for home. Gretchen releases the ball at an initial velocity of 75 ft/s at an angle of  $25^\circ$  with the horizontal. Assume Gretchen releases the ball 5 feet above the ground and aims it directly in line with the plate.

(a) Write two parametric equations that represent the path of the ball.

$$x = 75 \cos(25^\circ)t$$

$$y = -16t^2 + 75 \sin(25^\circ)t + 5$$

(b) Use a calculator to graph the path of the ball. Sketch the graph shown on the screen.



(c) How far will the ball travel horizontally before hitting the ground?

HITS ground when  $-16t^2 + 75 \sin(25^\circ)t + 5 = 0$

$$t \approx 2.1278827 \text{ seconds}$$

$$x(2.1278827) \approx 144.639 \text{ feet}$$

(d) When will the ball hit the ground?

$$t \approx 2.128 \text{ seconds}$$

(e) What is the maximum height of the trajectory?

graph  $y = -16t^2 + 75 \sin(25^\circ)t + 5$  in Function mode

CALCULATE MAX =  $20.69781 \text{ feet}$

$$(.99051094, 20.69781)$$

(f) When will the object reach its maximum height?

$$.99051094 \text{ seconds}$$