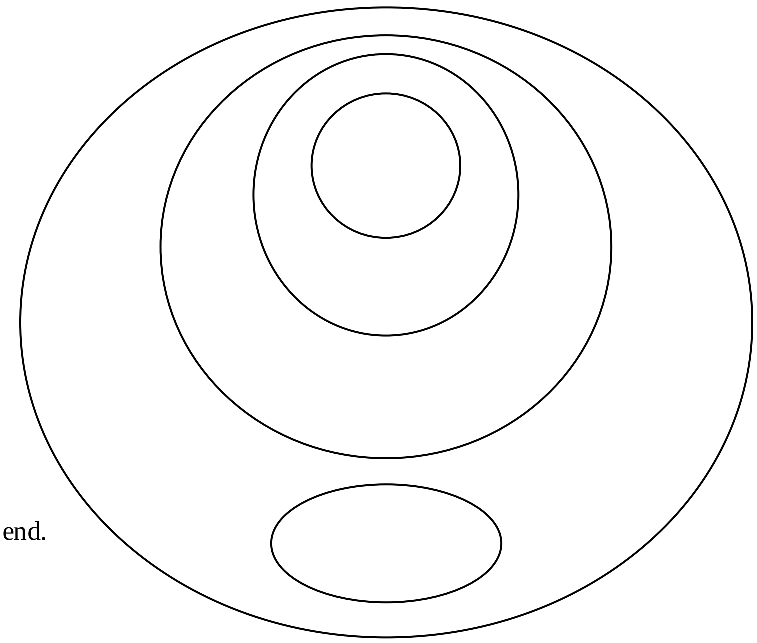
P.1 REAL NUMBERS

Learning Targets for P1

1. Describe an interval on the number line using inequalities
2. Describe an interval on the number line using interval notation (closed vs. open)
3. Switch between interval notation and inequality notation
4. Simplify exponential expressions
5. Identify Algebraic Properties (Commutative, Associative, Distributive, Identity, Inverse)

Types of Numbers



Natural (Counting) Numbers: ($\mathbb{N}$)

1, 2, 3, 4, …

Integers: ($\mathbb{Z}$)

…, –3, –2, –1, 0, 1, 2, 3, …

Rational Numbers: ($\mathbb{Q}$)

ALL fractions

ALL decimals that

a) repeat or

b) end

Irrational Numbers: … no symbol ($\mathbb{R} - \mathbb{Q}$)

Decimals that do NOT repeat AND do NOT end.

Real Numbers: ($\mathbb{R}$)

ALL Rational and Irrational Numbers

Inequalities and Intervals

Example 1: Graph the following on a number line:

a) $x \geq -2$ b) $5 < x$ c) $x > -8$

d) $-5 \leq x < 2$ e) $7 < x \leq 12$

Interval notation is another way to describe intervals instead of using an inequality.
The ends of an interval are either OPEN or CLOSED. Use a *one-sided parenthesis* to indicate the end of the interval *is open* and a *bracket* to indicate the end of the interval *is closed*.

Example 2: Fill in the blank

An ________________ interval is used when the endpoint IS NOT included.

A _________________ interval is used when the endpoint IS included.

Example 3: Go back to the last example and write the interval notation for *a* – *e*.

Example 4: Graph the following and write the corresponding inequality.

a) (3, 5] b) $[-12, \infty)$ c) $(-\infty, 7)$

Example 5: Use words to describe each of the intervals in the last example.

a) b) c)

Example 6: Use interval notation AND inequality notation to describe each interval on the *x* – axes below.

Example 7: Use both inequality and interval notation to describe the set of numbers. Define any variables used.

a) My teacher is at least 25 years old.

b) A grade of *A* in PreCalculus varies between 89.5% and 100%, including both these values.

<u>*Simplifying Expressions with Exponents*</u>

Example 8: Identify the base in each of the following expressions.

a) -4^2 b) $(-4)^2$ c) $(-3)^3$ d) -3^3

e) $2x+1^2$ f) $(2x+1)^2$

Example 9: Simplify each of the expressions in the last example.

<u>*Properties of Exponents*</u>

1. $u^m u^n =$
2. $\dfrac{u^m}{u^n} =$
3. $u^0 =$
4. $u^{-m} =$
5. $(uv)^m =$
6. $(u^m)^n =$
7. $\left(\dfrac{u}{v}\right)^m =$

Example 10: Simplify each of the following expressions.

a) $2x^{-1}$ b) $\left(\dfrac{3}{xy}\right)^{-2}$ c) $\left(\dfrac{3a^2b}{2a^3b}\right)\left(\dfrac{4b^3}{a^4b^2}\right)$ d) $\dfrac{(x^{-2}y^3)^{-2}}{(y^5x^{-2})^{-1}}$

<u>*Properties of Algebra*</u>

Commutative Property

Associative Property

Inverse Property

Identity Property

Distributive Property

Example 11: Complete the group activities on page 11, #45 and #46.

Using the Distributive Property

Example 12: Simplify each expression.

a) $\left(4xy+2x^3\right)ax^2$

b) $3a\left(2a+5b\right)$

Many times we need to undo the distributive property. This is the beginning of factoring. When you undo the distributive property, we say you have **factored out the greatest common factor**.

Example 13: Factor out the GCF from each expression.

a) $3x^2-6xy$

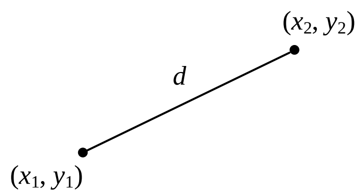
b) $15x^3-12x^2+9x$

P.2 CARTESIAN COORDINATE SYSTEM

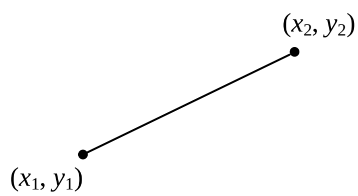
Learning Targets for P.2

1. Know and be able to use the distance formula
2. Know and be able to use the midpoint formula
3. Be able to identify a given equation as a circle
4. Be able to write the equation of a circle centered at (h, k) with a radius of r

Example 1: The distance formula is derived directly from the Pythagorean Theorem. Create a right triangle with the segment below, and solve for d.



Example 2: To find the midpoint of a segment simply find the average x and y-values of the endpoints.

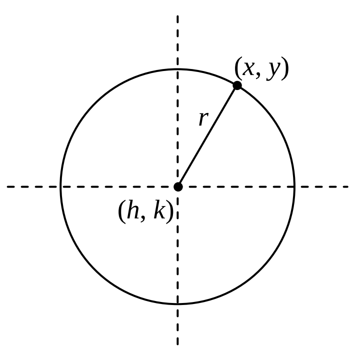


Example 3: Let the endpoints of a segment be (–2, 8) and (5, 7).

a) Find the length of the segment.

b) Find the midpoint of the segment.

Example 4: An equation of a circle whose center is at (h, k) can be written using the Pythagorean Theorem as well.



Example 5: Write an equation of the circle with radius 4 and whose center is at (0, –3).

Example 6: Sketch a graph of the equation $(x+1)^2 + (y-4)^2 = 9$.

P.3 LINEAR EQUATIONS AND INEQUALITIES

Learning Targets for P.3
1. Solve multiple step linear equations
2. Solve multiple step linear inequalities
3. Solve equations with fractions

Solving Linear Equations

Example 1: Solve each of the following equations..

a) $-3x - 8 = 4$

b) $5 - 2(h + 4) = 3h - 5$

c) $2(3 - 4b) - 5(2b + 3) = b - 17$

d) $\frac{x+3}{4} - \frac{x-1}{6} = 1$

e) $\frac{x-5}{15} + 4 = \frac{2x+1}{25}$

Solving Linear Inequalities

Example 2: Solve each inequality, graph the solution on a number line, and write the answer in interval notation.

a) $-3x - 8 < 4$

b) $\frac{-2x+5}{4} \leq 1$

c) $\frac{2y-3}{2} + \frac{3y-1}{5} > y - 1$

d) $\frac{1}{4}(x-4) - x \geq \frac{5}{2}(3-x)$

Example 3: Solve the following inequality and graph the solution on a number line:

$$-2 \le 3x + 4 < 5$$

<u>*Extending the Ideas*</u>

Example 4: The formula for the perimeter of a rectangle is given by $P = 2L + 2W$. Solve this equation for W.

Example 5: The formula for the area of a trapezoid is given by $A = \frac{1}{2}h\left(b_1 + b_2\right)$. Solve this equation for b_1.

Example 6: The formula for finding the Celsius temperature given the Fahrenheit temperature is $C = \frac{5}{9}\left(F - 32\right)$. Solve this equation for F.

P.4 LINES IN THE PLANE

Learning Targets for P.4

1. Calculate Average Rate of Change between 2 points
2. Write and graph an equation of a line in point-slope form
3. Write and graph an equation of a line in slope-intercept form
4. Write and graph an equation of a line in general form
5. Write and graph equations of horizontal and vertical lines
6. Understand how the slopes of parallel lines are related
7. Understand how the slopes of perpendicular lines are related

Slope

The slope of a non-vertical line is given by $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

A vertical line has ______________________, and a horizontal line has ______________________ .

Parallel Lines have slopes that are ________________ .

Perpendicular Lines have slopes that are __________________________________.

IMPORTANT ♬: You will be best served in this class if you think of slope as a _____________________________.

Equations of a Line

The first equation of a line you used in algebra was probably the *slope – intercept form*: _________________________

The slope is ______, and the *y*-intercept is ________.

In precalculus, it is actually easier to write the equation of a line in *point – slope form*: _________________________

The point is ________, and the slope is __________.

♬: To write an equation of a line, all you need is a _________________ and the _______________.

Another format used to write the equation of a line is called *standard (general) form*: ___________________________

Example 1: Which of the equations above has "*y* written as a function of *x*" ?

Example 2: The point-slope form is written as ___________________________ if you want "*y* written as a function of *x*"

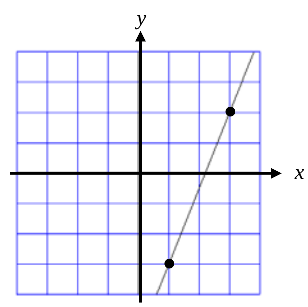
Example 3: For each of the following, write the equation of the line with the given information in point-slope form.

a) Point (–2, 6); Slope = –1

b) Point (1, –3); Slope = $\frac{5}{6}$

c) Points (12, 0) and (6, 3) are on the line.

d)



Example 4: Find the slope-intercept form of the line passing through (–2, 4) and having the following characteristics:

a) Slope of $\frac{7}{16}$

b) Parallel to the line $5x - 3y = 3$

c) Passing through the origin

d) Parallel to the y – axis.

Example 5: Find the general form of the line passing through (1, 3) and having the following characteristics:

a) Slope of $-\frac{2}{3}$

b) Perpendicular to the line $x + 2y = 0$

c) Passing through the point (2, 4)

d) Parallel to the x – axis.

Example 6: Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at (3, 4).

Example 7: The relationship between Fahrenheit and Celsius temperatures in linear.

a) Use the facts that water freezes at 0° C or 32° F, and water boils at 100° C or 212° F (not your recollection of temperature formulas) to find an equation that relates Celsius and Fahrenheit.

b) Using your equation, find the Celsius equivalent of 80° F and the Fahrenheit equivalent of –10° C.

c) Is there a temperature at which a Fahrenheit thermometer and a Celsius thermometer give the same reading? If so, what is it?

Example 8: Find the value of x or y so that the line through the pair of points has the given slope.

a) Points (x, 2) and (4, 8) with slope = 2.

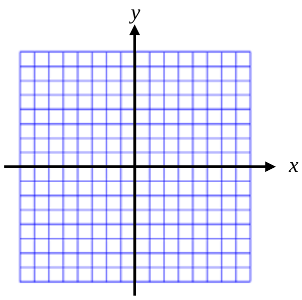
b) Points (–1, 3) and (4, y) with slope = ½ .

Example 9: At this point, we have covered 5 types of linear equations:

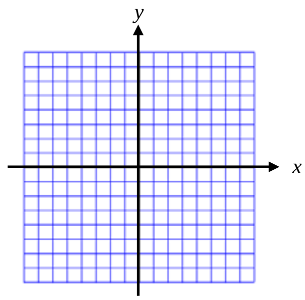
Horizontal Line
Point – Slope Form
Slope – Intercept Form
Standard Form
Vertical Line

Identify which form each equation below is in and graph each linear equation on the graph paper provided.

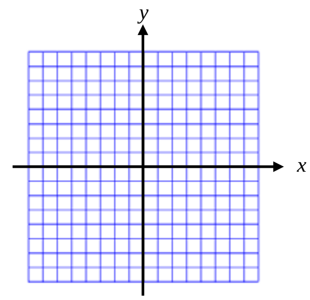
a) $y-2=3(x-1)$



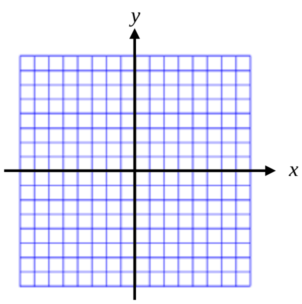
b) $y=3x+4$



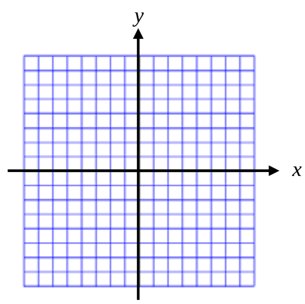
c) $2x+3y=6$



d) $y=7$



e) $x=-2$



FACTORING

Learning Targets for Factoring

1. Factor out a GCF
2. Factor difference of squares
3. Factor a quadratic expression of the form $ax^2 + bx + c$
4. Factor an expression by grouping

Greatest Common Factor (GCF)

The first step in factoring is to factor out a GCF. We did this in P.1.

Example 1: Factor out the GCF from each expression.

a) $3x^2 + 6x$

b) $5x^4 - 7x^3 + 2x^2$

Factoring Quadratic Expressions of the Form $ax^2 + bx + c$

Watch the PowerPoint Tutorial from http://www.chaoticgolf.com/tutorials on Factoring Quadratic Expressions

Example 2: Factor each expression completely.

a) $3x^2 - 4x - 7$

b) $2x^2 + 11x + 5$

c) $6x^2 - 2x - 8$

d) $6x^2 - 19x + 15$

Difference of Two Perfect Squares

Example 3: Factor each expression.

a) $a^2 - b^2$

b) $9x^2 - 25y^2$

c) $3x^2 - 16$

d) $36x^2 + 49$

Example 4: Completely factor each expression.

a) $4x^4 + 24x^3 + 32x^2$.

b) $3(2a-3)^2 + 17(2a-3) + 10$

P.5 SOLVING EQUATIONS GRAPHICALLY, NUMERICALLY, AND ALGEBRAICALLY

Learning Targets for P.5

1. Solve equations graphically with a calculator (2 ways)
2. Solve quadratic equations algebraically: x^2 but no x, x^2 and x but no c, x^2 and x and c, undo $(\ldots)^2$
3. Complete the square
4. Solve Absolute value equations

Being able to solve equations in multiple ways allows you to answer a question using the easiest method possible. Another advantage of having multiple ways to solve an equation is that it gives you an opportunity to check your work.

The three methods are obvious from the title of the section … Graphically, Numerically, and Algebraically.

Solve an Equation by Graphing

There are two keys to solving an equation by graphing.

1. Understanding WHERE the solution(s) is(are).
2. Finding those solutions using your calculator (or a hand drawn graph).

Solutions are located in _________ possible places, either the _____________________or _________________________.

Example 1: Solve the equation $2x^2 - 5x = 3$ by graphing the left side of the equation and the right side of the equation.

a) Where is(are) the solution(s) to this equation located?

b) What is the solution to this equation?

Example 2: Solve the equation $2x^2 - 5x = 3$ by setting one side equal to 0, and graphing the other side.

a) Where is(are) the solution(s) to this equation located?

b) What is the solution to this equation?

Example 3: Graphically solve the equation $x^3 + x^2 + 2x - 3 = 0$. Describe your method.

Example 4: Solve the equation $2^x = x^2$ graphically.

a) Solve by finding the intersections.

b) Solve by finding the zeros.

Solve an Equation Numerically

Typically, you will be given a table of values when asked to solve an equation numerically. You could also create the table yourself.

Zeros occur when there is a _____________ change in the _________ values in the table.

The *zero* is between the _________ values in the table, where the __________________ occurs.

Example 5: Use the table to approximate the zero of the function, $y = -x^3 + x + 1$.

x	y
1.1	0.769
1.2	0.472
1.3	0.103
1.4	– 0.344
1.5	– 0.875
1.6	– 1.496
1.7	– 2.213

Solve an Equation Algebraically

This is the method you have spent the most time using in math classes so far. We are going to focus on various methods of solving quadratic equations on these notes.

Example 6: Method 1: Solve by extracting square roots.

a) $(2y+3)^2 = 16$

b) $2(x-4)^2 = 10$

Example 7: Method 2: Solve by completing the square.

a) $x^2 + 6x = 7$

b) $2x^2 + 5x - 12 = 0$

Example 8: Method 3: Solve using the quadratic formula.

a) $2x^2 - x - 3 = 0$

b) $3x^2 - 6x - 7 = x^2 + 3x - x(x+1) + 3$

Example 9: Method 4: Solve by factoring. Support your work graphically.

a) $x^2 - x - 6 = 0$

b) $4x^2 + 3 = 8x$

P.6 SOLVING INEQUALITIES ALGEBRAICALLY AND GRAPHICALLY

Learning Targets for P.6

1. Solve Absolute Value Inequalities using correct notation and vocabulary
2. Solve quadratic or cubic inequalities by finding zeros on a graph
3. Apply solving inequalities to context including but not limited to projectile motion

Solving inequalities will be much like solving equations. However, when absolute values are involved, there are a few definitions and rules you must remember.

Definition of Absolute Value

If x is any real number, then the absolute value of x, denoted $|x|$ is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Absolute Value is best viewed as a distance. When finding the absolute value of a single number, you are finding the

_______________________ that number is from _______________.

Example 1: Write an absolute value equation that expresses the following statement:

"The distance between x and the number 4 is 7"

Example 2: Consider the equation $|2x+5| = 9$.

a) Solve the equation algebraically.

b) Solve the equation graphically. Describe your method.

c) If we factor out a 2 from the left side of the expression we get $2\left|x+\frac{5}{2}\right| = 9$. We can then divide both sides by 2 to obtain $\left|x+\frac{5}{2}\right| = \frac{9}{2}$. Using the concept of absolute value as a distance, draw the solution to this equation on a number line.

<u>*Rules to Remember When Solving Absolute Value Inequalities*</u>

Let u be an algebraic expression in x and let a be a real number greater than 0.

1. $|u| < a$ if and only if $-a < u < a$

2. $|u| > a$ if and only if $u < -a$ or $u > a$

Informally, what these means is if you are represented by , then || represents your distance from zero.

Example 3: If || > 5, use a number line to represent where YOU are allowed to be.

Example 4: If || < 5, use a number line to represent where YOU are allowed to be.

Example 5: Solve each inequality algebraically. Write your solution in interval notation.

a) $|2x-1| > 35$

b) $|3-4x| + 2 \le 9$

<u>*Solving Inequalities Without Absolute Values … USING SIGN CHARTS!!!!!*</u>

We will spend much more time with this later on in the year, but for now, a quick introduction.

Example 6: Solve $x^2 + x - 6 < 0$

a) Graphically

b) Algebraically using the ZEROS and a SIGN CHART

Example 7: The height , *h*, in feet of a projectile *t* seconds after is has been launched vertically from an initial height of h_0 feet above the ground with an initial velocity of v_0 feet/second is given by the formula
$h = -16t^2 + v_0 t + h_0$

a) If an object is launched with an initial velocity of 256 ft/sec from the ground, write the equation modeling the height of the object.

b) When does the object hit the ground?

c) When does the object reach it's maximum height?

d) What is the maximum height of the object?

e) When will this object be at least 768 feet above the ground?

2.8 SOLVING RATIONAL EQUATIONS

Learning Targets for 2.8

1. Be able to find the LCD of 3 polynomials
2. Solve a Rational Equation by clearing the fractions
3. Understand when a solution to a rational equation is extraneous and identify them when found

Example: When we solved equations involving fractions, we eliminated the fractions by

Example: Find the least common denominator of the following pairs of fractions:

a) $\frac{1}{3t}$ and $\frac{1}{5t^2}$

b) $\frac{x}{8}$ and $\frac{3x}{2}$

c) $\frac{4}{3h^2}$ and $\frac{2}{h^3}$

d) $\frac{4}{2z^2+2z}$ and $\frac{5z}{z+1}$

e) $\frac{4}{y+2}$ and $\frac{3+y}{y-1}$

f) $\frac{1}{k+2}$ and $\frac{3k}{k^2-4}$

g) $\frac{7x-6}{x^2-7x-8}$ and $\frac{3x}{2x^2-19x+24}$

Example: Solve the following equations.

a) $\frac{a}{3}+\frac{a}{5}=4$

b) $\frac{5}{4x}+\frac{2}{3}=\frac{1}{x}$

Finding the LEAST common denominator will help you eliminate the fractions in an equation. However, when you multiply both sides of an equation by an algebraic expression you create the possibility of *extraneous solutions*. Extraneous solutions are answers that you will get by following all the correct steps, but they do not work in the original equation. For the purposes of this lesson, usually this occurs when you find an answer that creates a 0 in the denominator of the original problem.

Example: Solve the following equations. Check for extraneous solutions.

a) $\frac{3}{x+2}=\frac{4}{x-3}$

b) $\frac{5}{2x-2}=\frac{15}{x^2-1}$

c) $\frac{2}{x-3}-\frac{4}{x+3}=\frac{8}{x^2-9}$

d) $\frac{n}{n^2+2n}+\frac{1}{n}=\frac{3}{n+2}$

e) $\frac{3x}{x+5}+\frac{1}{x-2}=\frac{7}{x^2+3x-10}$

f) $\frac{2}{x-3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$