PreCalculus
Chapter 5 Review

Name: KEY
Block: _____

For questions 1 – 3, use identities and/or factoring to simplify each expression.

1. $\dfrac{\sin^2\beta\cot\beta}{\cos\beta}$

$\sin^2\beta \cdot \dfrac{\cos\beta}{\sin\beta} \cdot \dfrac{1}{\cos\beta}$

$\boxed{\sin\beta}$

2. $\sin(x) - \sin(x)\cos^2(x)$

$\sin x\left(1-\cos^2 x\right)$

$\sin x\left(\sin^2 x\right)$

$\boxed{\sin^3 x}$

3. $\sin^2 x + \cos\left(\dfrac{\pi}{2} - x\right) - 1 + \cos^2 x$

$\sin^2 x + \cos(\pi/2 - x) - 1$

$\sin x$

For questions 4 – 8, verify that each of the following is an identity.

4. $\dfrac{\cos^2\theta}{\sin^2\theta} + \csc\theta\sin\theta = \csc^2\theta$

$= \cot^2\theta + 1$

$= \boxed{\csc^2\theta}$

5. $\dfrac{1}{1-\sin x} + \dfrac{1}{1+\sin x} = 2\sec^2 x$

$= \dfrac{1}{(1-\sin x)}\dfrac{(1+\sin x)}{(1+\sin x)} + \dfrac{1}{(1+\sin x)}\dfrac{(1-\sin x)}{(1-\sin x)}$

$= \dfrac{(1+\sin x) + (1-\sin x)}{1-\sin^2 x}$

$= \dfrac{2}{\cos^2 x} = \boxed{2\sec^2 x}$

6. $\dfrac{1+\cos 2\alpha}{\sin 2\alpha} = \cot\alpha$

$\dfrac{1+(2\cos^2\alpha - 1)}{\sin 2\alpha}$

$\dfrac{2\cos^2\alpha}{2\sin\alpha\cos\alpha} = \dfrac{\cos\alpha}{\sin\alpha} = \boxed{\cot\alpha}$

7. $\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

THIS IS A COFUNCTION IDENTITY PROVE IT USING SUM/DIFFERENCE IDENTITY!

$\cos\left(\frac{\pi}{2}\right)\cos(x) + \sin\left(\frac{\pi}{2}\right)\sin x$

$0 \qquad 1$

$0 + \sin x$

$\boxed{\sin x}$ ✓

8. $\sin\left(\dfrac{3\pi}{2} + x\right) = -\cos x$

NOT A COFUNCTION IDENTITY since it's $3\pi/2$ instead of $\pi/2$!

$\sin\left(\frac{3\pi}{2}\right)\cos(x) + \cos\left(3\pi/2\right)\cdot\sin(x) = -1\cdot\cos x + 0 = -\cos x$

$-1 \qquad 0$

For questions 9 – 13, use identities to find the exact value of each function.

9. $\sin(105°)$

$\sin(60+45)$

$\sin 60\cos 45 + \cos 60\sin 45$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

10. $\tan\left(-\frac{\pi}{12}\right) = \tan(-15°)$

$\tan(45-60) = \frac{\tan(45)-\tan(60)}{1+\tan(45)\cdot\tan(60)}$

$= \frac{1-\sqrt{3}}{1+(1)(\sqrt{3})} = \boxed{\frac{1-\sqrt{3}}{1+\sqrt{3}}}$ or $\boxed{-2+\sqrt{3}}$

11. $\cos(50°)\cos(20°) + \sin(50°)\sin(20°)$

$= \cos(50-20)$

$= \cos(30°) = \boxed{\frac{\sqrt{3}}{2}}$

12. $1 - 2\sin^2\left(\frac{\pi}{8}\right)$

$= \cos\left(2\cdot\frac{\pi}{8}\right)$

$= \cos(\pi/4) = \boxed{\frac{\sqrt{2}}{2}}$ or $\boxed{\frac{1}{\sqrt{2}}}$

13. $\frac{2\tan 75°}{1-\tan^2 75°} = \tan(2\cdot 75°)$

$= \tan(150°)$

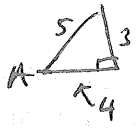
$= \boxed{-\frac{1}{\sqrt{3}}}$ or $\boxed{\frac{-\sqrt{3}}{3}}$

For questions 14 – 16, if $\sin A = \frac{3}{5}$ and A is in the second quadrant, find each value.

means sine is +, cosine is −, & tangent is −

$\cos A = -4/5 \quad \tan A = -3/4$

14. $\cos(2A)$



$1 - 2\sin^2(A)$

$1 - 2(3/5)^2 = 1 - 2(9/25) = 1 - \frac{18}{25} = \boxed{\frac{7}{25}}$

15. $\sin(2A)$

$= 2\sin A\cos A$

$= 2(3/5)(-4/5) = \boxed{-\frac{24}{25}}$

16. $\tan(2A)$

$\frac{2\tan A}{1-\tan^2 A} = \frac{2(-3/4)}{1-(-3/4)^2} = \frac{-3/2}{1-\frac{9}{16}}$

$= \frac{-3/2}{7/16} = -\frac{3}{2}\cdot\frac{16}{7} = \boxed{-\frac{24}{7}}$

For questions 17 – 20, solve each equation for [0, 2π).

17. $2\cos^2(x) - 5\cos(x) + 3 = 0$

$(2\cos x - 3)(\cos x - 1) = 0$

$\cos x = \frac{3}{2}$ not possible

$\cos x = 1$

$\boxed{x = 0}$

18. $\cos^2(x) + 4\sin(x) + 4 = 0$

$(1-\sin^2 x) + 4\sin x + 4 = 0$

$5 + 4\sin x - \sin^2 x = 0$

$(5 - \sin x)(1 + \sin x) = 0$

$\sin x = 5$ not possible

$\sin x = -1$

$\boxed{x = 3\pi/2}$

19. $\cos(2x) - 2\sin^2(x) = 0$

$(1 - 2\sin^2 x) - 2\sin^2 x = 0$

$1 - 4\sin^2 x = 0$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$\boxed{x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6}$

20. $4\cos^2(x) - 3 = 0$

$4\cos^2 x = 3$

$\cos^2 x = 3/4$

$\cos x = \pm\sqrt{3}/2$

$\boxed{x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6}$

For question 21, solve for all values.

21. $2\tan(x)\sin(x) + \tan(x) = 0$

$\tan x(2\sin x + 1) = 0$

$\tan x = 0 \qquad \sin x = -1/2$

$x = 0, \pi \qquad x = 7\pi/6 \text{ or } 11\pi/6$

$x = 0 + 2\pi \cdot k$

$x = \pi + 2\pi \cdot k$

$x = \frac{7\pi}{6} + 2\pi \cdot k$

$x = \frac{11\pi}{6} + 2\pi \cdot k$

where k is any integer

For questions 22 and 23, find the area of each triangle to the nearest tenth.

22. $a = 5, b = 12, c = 13$ (SSS)

HERON'S $\quad s = \frac{5+12+13}{2} = 15$

$A = \sqrt{15(15-5)(15-12)(15-13)}$

$= \sqrt{15(10)(3)(2)} = \sqrt{900} = 30$ square units

23. $c = 3.58, b = 6.8, A = 39°$ (SAS)

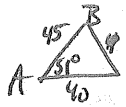
$A = \frac{1}{2}(6.8)(3.58)\sin(39°)$

$A \approx 7.660$ square units

For questions 24 and 25, solve each triangle. Round your FINAL answer to the nearest tenth.

(SAS)

24. $b = 40, c = 45, A = 51°$



$a^2 = (40)^2 + (45)^2 - 2(40)(45)\cos(51°)$

$a = 36.87067388$ (SAME AS "A")

$a \approx 36.9$

$\angle B \approx 57.5°$

$\angle C \approx 71.5°$

$40^2 = a^2 + 45^2 - 2a(45)\cos(B)$

$.5377494435 = \cos B \Rightarrow B \approx 57.46943527° \Rightarrow \angle C = 180 - \angle A - \angle B \approx 71.53056473°$

25. $c = 125, b = 150, C = 25°$

* see next page

Previous chapter review

For questions 26 – 31, find each EXACT value.

26. $\sin\left(\frac{5\pi}{6}\right) = 1/2$

27. $\cos\left(-\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ or $-\frac{1}{\sqrt{2}}$

28. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

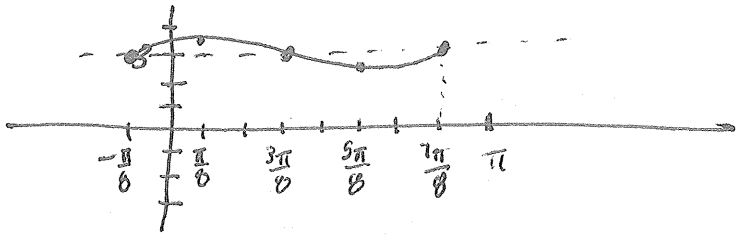
29. $\tan^{-1}(0) = 0$

30. $\tan\left|\arcsin\left(\frac{\sqrt{3}}{2}\right)\right| = \sqrt{3}$

31. $\cos\left|\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right| = \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$

32. Graph: $y = \frac{1}{2}\sin 2\left(x + \frac{\pi}{8}\right) + 3$. Label axes. State the amplitude, period, phase shift, and vertical shift.

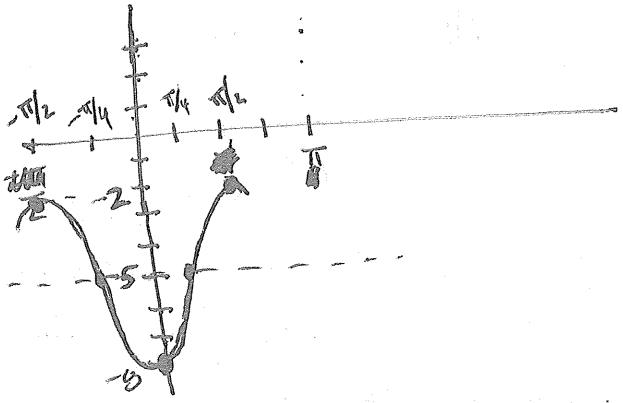
Amp = 1/2
Period = $2\pi/2 = \pi$
Phase Shift = Left $\pi/8$
Vertical Shift = Up 3



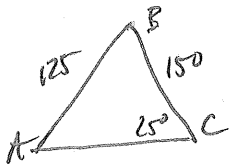
33. Write an equation using the cosine function from the given information. Sketch a graph of the curve.

amplitude = 3, period = π, phase shift = $-\frac{\pi}{2}$, vertical shift = –5.

$y = 3\cos\left(2\left(x + \pi/2\right)\right) - 5$



#25 c = 125
b = 150
C = 25°



"ASS"

$\frac{\sin A}{150} = \frac{\sin 25}{125}$

$\sin A = \frac{150 \sin 25}{125}$

If
$\angle A \approx 30.4736409$ ↑ store as "A"
OR
$\angle A \approx 149.5263591°$ ↑ store as "D"

if C = 25°, then
$\angle B = 124.5263591°$ ↑ store as "B"

$\frac{\sin B}{b} = \frac{\sin 25}{125} \Rightarrow b = \frac{125 \sin B}{\sin 25} \approx 243.6789885$

2 As Possible

	OR	
$\angle A \approx 30.5°$		$\angle A \approx 149.5°$
$\angle B \approx 124.5°$		$\angle B \approx 5.5°$
$b \approx 243.7$		$b \approx 28.2$

if C = 25°, then
$\angle B = 5.473640898°$ ↑ store as "E"

which is really E

$\frac{\sin B}{b} = \frac{\sin 25}{125}$

$b = \frac{125 \sin(E)}{\sin(25)}$

$b \approx 28.21334757$