P.6 SOLVING INEQUALITIES ALGEBRAICALLY AND GRAPHICALLY

Solving inequalities will be much like solving equations. However, when absolute values are involved, there are a few definitions and rules you must remember.

Definition of Absolute Value

If x is any real number, then the absolute value of x, denoted $|x|$ is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Absolute Value is best viewed as a distance. When finding the absolute value of a single number, you are finding the _____ that number is from _____.

Example: Write an absolute value equation that expresses the following statement:
"The distance between x and the number 4 is 7"

Example: Consider the equation $|2x+5| = 9$.

a) Solve the equation algebraically.

b) Solve the equation graphically. Describe your method.

c) If we factor out a 2 from the left side of the expression we get $2\left|x+\frac{5}{2}\right| = 9$. We can then divide both sides by 2 to obtain $\left|x+\frac{5}{2}\right| = \frac{9}{2}$. Using the concept of absolute value as a distance, draw the solution to this equation on a number line.

Rules to Remember When Solving Absolute Value Inequalities

Let u be an algebraic expression in x and let a be a real number greater than 0.

1. $|u| < a$ if and only if $-a < u < a$
2. $|u| > a$ if and only if $u < -a$ or $u > a$

Example: Solve each inequality algebraically. Write your solution in interval notation.

a) $|2x-1| > 35$

b) $|3-4x|+2 \le 9$

Solving Inequalities Without Absolute Values

We will spend much time with this later on in the year, but for now, a quick introduction.

Example: Solve $x^2 + x - 6 < 0$ by finding the zeros of the function and using a sign chart.

Example: The height , h, in feet of a projectile t seconds after is has been launched vertically from an initial height of h_0 feet above the ground with an initial velocity of v_0 feet/second is given by the formula
$h = -16t^2 + v_0 t + h_0$

a) If an object is launched with an initial velocity of 256 ft/sec from the ground, write the equation modeling the height of the object.

b) When will this object be at least 768 feet above the ground?