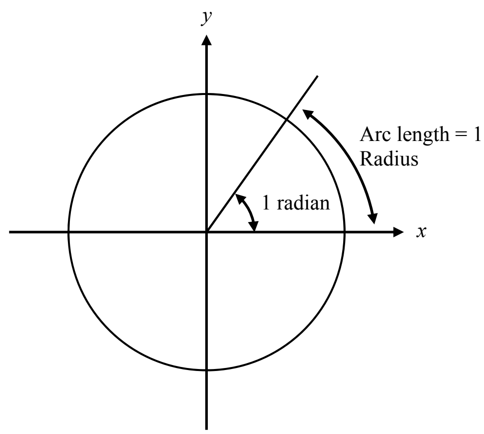
Learning Targets

1. Draw an angle in Radians by thinking in radians (without converting to degrees).
2. List the reference angle of the radian angle.
3. Convert Degrees → Radians and vice versa.
4. Apply arc length formulas in Radians when given a radian angle or a degree angle.
5. Convert linear speed to angular speed & vice versa
6. Relate definitions of bearing and heading to real world problems.
7. Convert DMS to decimal degrees and vice versa

**There are multiple ways to measure angles: degrees, revolutions, bearings and radians**

- **Degrees:** From Geometry: straight angle = 180 degrees and a circle = 360 degrees
- **Revolutions:** 1 revolution = 1 full turn around a circle
- **Radians:** An angle of **1 radian** is defined to be the angle at the center of a circle which spans an arc of length equal to the radius of that circle.




**Standard Position:** For degree angles, revolutions and radian angles, we draw the angles with their initial side along the positive $x$-axis of the coordinate axes. The terminal (end) side of the angle is then measured in a counterclockwise direction. NOTE: Bearings work differently and will be covered in class.

**Reference Angles:** The acute angle between the terminal side of an angle and the $x$-axis.

*Example 1:* Draw the following Radian angles. Then list their reference angle.

a) $\frac{\pi}{2}$ b) $\frac{11\pi}{6}$ c) $\frac{4\pi}{3}$ d) 2.9

*Example 2:* Convert from degrees to radians or radians to degrees:

a) $200°$

b) $\frac{7\pi}{8}$ radians

**ArcLength:** The length of a portion of the circumference of a circle is $S = \theta r$ where $S$ is the arc length, $\theta$ is the angle measured in radians, and $r$ is the radius of the circle.

*Example 3:* Find the length of an arc. Express the answer in terms of $\pi$

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a) $\theta = \frac{5\pi}{6}$; $r = 4$ cm

b) $\theta = 125°$; $r = 1.5$ mm

**Angular and Linear Motion**

- Angular Speed: How fast something is spinning.
- Linear Speed: How fast something is travelling in one direction.

*Example 4:* A turntable rotates at 50 revolutions per minute. What is its angular speed in radians per second?

Learning Targets

1. Know and apply the six trigonometric ratios
2. Solve right triangles using the six trig. ratios
3. Know the ratios of the sides of the 30-60-90 special right triangle
4. Know the ratios of the sides of the 45-45-90 special right triangle
5. Apply the ratios of the special right triangles to real life application questions.

*Next let's put our angles inside triangles…specifically right triangles.*

❖ **Three Basic Trigonometric Ratios**

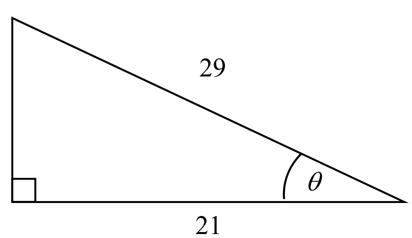
$$\sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}},\ \cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}},\ \tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta}$$

- Remember the ratios are used on an acute angle.
- We memorized _______________.

❖ **The Reciprocal Ratios: __________________, _________________ and ___________________**

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

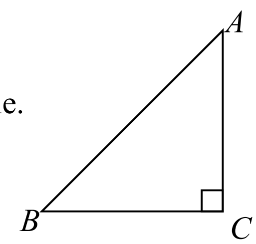
*Example 1:* Using the triangle at the right, find all six trigonometric functions of the angle θ.




*Example 2:* Given $\tan\theta = \frac{5}{12}$, find the remaining trigonometric functions.

***Special Right Triangles:***

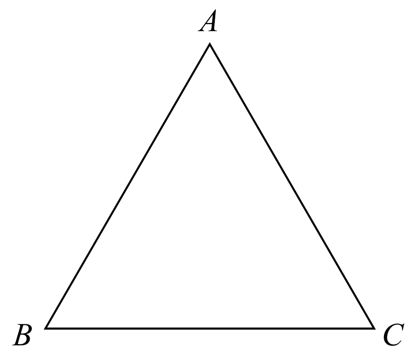
**#1: 45 – 45 – 90 Right Triangle (MEMORIZE THESE RATIOS)**




a) If $AC = 1$, and $m\measuredangle A = 45°$, solve for the remaining parts of the triangle.

b) Find the sine, cosine and tangent values of 45°.

**#2: 30 -60 – 90 Right Triangle (MEMORIZE THESE RATIOS)**




a) $\Delta ABC$ is equilateral, so each angle is ____________.

b) Draw the altitude of the triangle from $A$ to $\overline{BC}$.
Call the point of intersection $D$.

c) Therefore, $m\measuredangle BAD =$ ________.

d) Suppose $AB = 2$. Solve for the remaining parts of $\Delta BAD$.

e) Find the sine, cosine and tangent values of 30° and 60°.

*Example 4:* A ladder is extended to reach the top floor of an 84 foot tall burning building. The fire fighters see someone who needs rescuing in a window 8 feet below the roof. How far should the ladder be extended to reach the roof if the ladder must be placed at the optimum operating angle of 60°?

Learning Targets

1. Graph Radian and Degree angles in standard position
2. Find reference angles for Radian and Degree angles in standard position
3. Identify positive and negative angles that are Coterminal with a given angle.
4. Find the exact value of the six trig. ratios of an angle in standard position

Day 1

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5. Find the exact value of the six trig. ratios of quadrantal angles
6. Find the exact value of the six trig. ratios of non-quadrantal angles.

Day 2

**DAY 1:**

SohCahToa works well for acute angles; but what if we need angles 90° or larger?
What if we need negative angles?

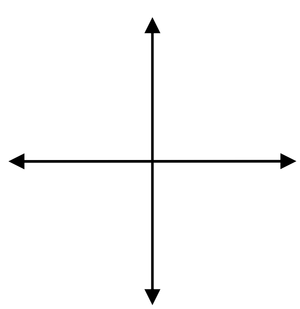
**Standard Position:** By placing angles in standard position, we can extend the terminal sides past the first quadrant. Remember each of the following facts…

- Positive Angles are measured counterclockwise.
- Negative Angles are measured clockwise.
- Reference Angles are still the acute angle measured to the *x*-axis (just as in lesson 4.1)

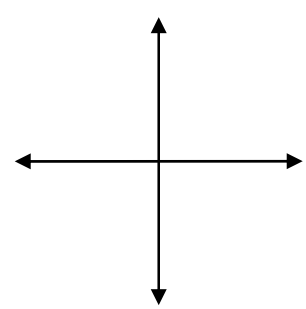
**Coterminal Angles:** Angles with the same initial and terminal sides, but different measures. The measures differ by integral multiple(s) of $2\pi$ or $360°$.

*Example1:* Find and draw one positive and one negative angle that is co terminal with the given angle.

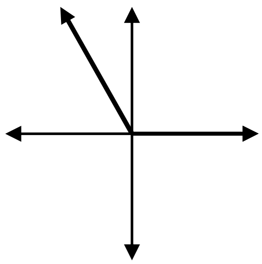
a) $-210°$



b) $\frac{13\pi}{4}$



**Trigonometric Functions – Redefined:** Let $\theta$ be any angle in standard position and let $P(x, y)$ be any point on the terminal side of the angle. Then $r$ is the distance from the origin to $P(x, y)$ or the radius of a circle and $r = \sqrt{x^2 + y^2}$ . The six Trigonometric Ratios are…



$\sin\theta =$ $\qquad$ $\cos\theta =$ $\qquad$ $\tan\theta =$

$\csc\theta =$ $\qquad$ $\sec\theta =$ $\qquad$ $\cot\theta =$

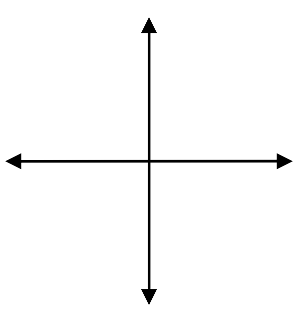
→ Notice if $r = 1$, then we have a "Unit Circle" which is a circle with radius 1.

$\sin\theta =$ $\qquad$ $\cos\theta =$ $\qquad$ $\tan\theta =$

*Example 2:* Find the six trigonometric ratios of $\theta$ whose terminal side passes through the given point.

a) $(3, 4)$ $\qquad$ b) $(3, -4)$

**A Shortcut for the Signs of Trigonometric Functions:**



Now we know the trig functions when given a point. We can also predict the _________ of the trig functions if we know the ______________. But how do we find the trig functions when given an angle?

**DAY 2:**

**Quadrantal Angles:** Any angle in standard position whose terminal side is on the $x$-axis or the $y$-axis.

*Example 3:* Find the exact value of each given trig function using the given angle.

a) $\sin \pi$

b) $\cos(-360°)$

c) $\csc\left(\frac{3\pi}{2}\right)$

d) $\sec\left(\frac{-11\pi}{2}\right)$

*Remember that you cannot divide by zero, so sometimes* ________________ *&* ____________________ *are undefined!*

**Non – Quadrantal Angles:** Any angle in standard position whose terminal side is NOT on the $x$-axis or the $y$-axis. To find a trig. ratio for a non-quadrantal angle, we need three things:

1. ____________________: We identify the quadrant of the angle according to its terminal side. The quadrant tells us the sign of the trigonometric ratio.

2. ____________________________: Reference triangles are right triangles created by drawing a perpendicular from the terminal side of a non-quadrantal angle to the $x$-axis. The reference angles we used in Lesson 4.1 are inside this right triangle and for our purposes here, the reference angle will always be a multiple of the angles found in the special right triangles from Lesson 4.2.

3. ________________________________: Remember we memorized these ratios (or the sine, cosine and tangent of these angles) in lesson 4.2.

*Example 4:* Draw the given angle and list the quadrant in which it lies. List the measure of $\theta_{ref}$ . Then, evaluate the indicated trig value.

a) $\csc(-60°)$

b) $\cos\left(\frac{7\pi}{4}\right)$

c) $\sin\left(\frac{-5\pi}{6}\right)$

d) $\tan 120°$

*Now you have all the pieces to solve any puzzle…*

*Example 5:* Find $\cos\theta$ and $\cot\theta$ if $\sin\theta = \frac{1}{4}$ and $\tan\theta < 0$.