

2.4 REAL ZEROS OF POLYNOMIAL FUNCTIONS

In the last section, we found the zeros of a polynomial by factoring. In this section we are going to explore different ways to obtain the zeros, starting with long division. The use of long division (and synthetic division) will aid us in our attempts to factor the polynomials.

Example: Divide. Write a summary statement in polynomial form.

a) $x+4 \overline{)x^3 + 2x^2 - 5x + 6}$

b) $2x+1 \overline{)2x^3 - 3x^2 + 6x - 7}$

summary: _____

summary: _____

3. $\frac{2x^4 - 3x^3 + 5x - 1}{x - 2}$

summary: _____

As long as you are dividing by a binomial term of the form $x - c$, you can use synthetic division.

Example: Divide using synthetic division.

a) $(x^3 + 2x^2 - 5x + 6) \div (x + 4)$

b) $(2x^4 - 3x^3 + 5x - 1) \div (x - 2)$

Remainder Theorem:

Factor Theorem:

Example: Find the remainder for each division. Verify using synthetic division.

a) $(2x^3 - 3x^2 + x) \div (x-1)$

b) $(x^5 - 2x^4 + x - 9) \div (x+2)$

Example: Use the factor theorem to determine whether the first polynomial is a factor of the second.

a) $x - 1$; $x^3 - x^2 + x - 1$

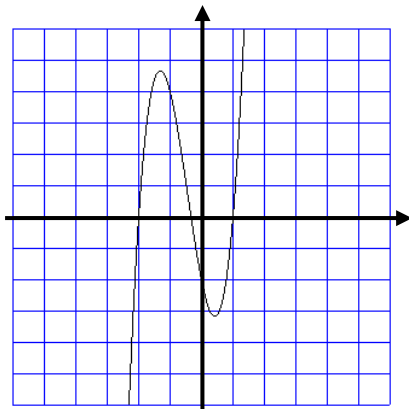
b) $x + 2$; $4x^3 + 9x^2 - 3x - 7$

Example: Find the polynomial function in the form: $y = a(x - b)(x - c)(x - d)$.

x	1	3	-4	-1
y	0	0	0	2

Example: Use the graph to completely factor $f(x)$ with the aid of synthetic division.

$$f(x) = 3x^3 + 4x^2 - 5x - 2$$



Rational Zeros Theorem: Let $f(x)$ be a polynomial function with integral coefficients. Then, the only possible rational zeros of $f(x)$ are: $\frac{p}{q}$ where p is a divisor of the constant term and q is a divisor of the leading coefficient.

List all possible rational zeros of the function: $f(x) = 2x^3 - 5x^2 - 4x + 3$

Possible values of p :

Possible values of q :

Possible rational roots $\frac{p}{q}$:

Graph the function to see which **are** the zeros of $f(x)$:

Use synthetic substitution and/or the quadratic formula to find the other two real zeros.