

2.4 REAL ZEROS OF POLYNOMIAL FUNCTIONS**Learning Targets:**

1. Use long division to find factors of a polynomial.
2. Use synthetic division to find linear factors of a polynomial.
3. Apply the remainder theorem to find the function value at a given value of x .
4. Apply the factor theorem to write the factors of a polynomial.

In the last section, we found the zeros of a polynomial by factoring. In this section we are going to explore different ways to obtain those factors. We will focus on long division and synthetic division.

Example 1: Divide. Write a summary statement in polynomial form.

a) $x+4 \overline{)x^3 + 2x^2 - 5x + 6}$

b) $2x+1 \overline{)2x^3 - 3x^2 + 6x - 7}$

summary: _____

summary: _____

c) $\frac{2x^4 - 3x^3 + 5x - 1}{x - 2}$

summary: _____

As long as you are dividing by a term that looks like _____, you can use synthetic division.

Example 2: Divide using synthetic division.

a) $(x^3 + 2x^2 - 5x + 6) \div (x + 4)$

b) $(2x^4 - 3x^3 + 5x - 1) \div (x - 2)$

The Remainder Theorem

If a polynomial is divided by $(x - c)$, then the remainder is the same as the function value $f(c)$.

Example 3: Use the remainder theorem to find the remainder. Verify using synthetic division.

a) $(2x^3 - 3x^2 + x) \div (x - 1)$

b) $(x^5 - 2x^4 + x - 9) \div (x + 2)$

The Factor Theorem

For a polynomial function, $x = c$ is a zero of a function if and only if $(x - c)$ is a factor of the function.

Example 4: Use the factor theorem to determine whether the first polynomial is a factor of the second.

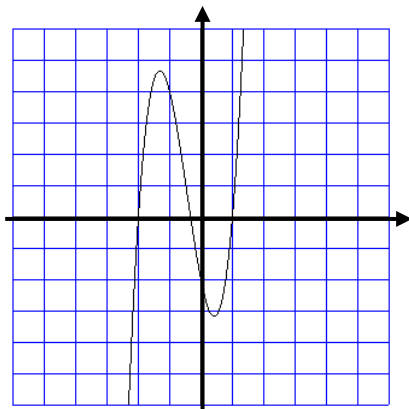
a) $x - 1$; $x^3 - x^2 + x - 1$

b) $x + 2$; $4x^3 + 9x^2 - 3x - 7$

Example 5: Find the polynomial function in the form: $y = a(x - b)(x - c)(x - d)$.

x	1	3	-4	-1
y	0	0	0	2

Example 6: Completely factor $f(x) = 3x^3 + 4x^2 - 5x - 2$. Use the graph below and synthetic division.



A Quick Summary ...

The following statements are all equivalent:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$.
2. k is a zero of the function $f(x)$.
3. k is an x -intercept of the graph of $f(x)$.
4. $(x - k)$ is a factor of $f(x)$.

Rational Zeros Theorem

Let $f(x)$ be a polynomial function with integral coefficients.

The only *possible rational zeros* of $f(x)$ are:

$$\frac{p}{q},$$

where p is a divisor of the constant term and q is a divisor of the leading coefficient.

Example 7: List all possible rational zeros of the function: $f(x) = 6x^3 + 5x^2 - 21x + 10$

- a) List all possible values of p (divisors of 10).
- b) List all possible values of q (divisors of 6).
- c) List all possible rational roots $\frac{p}{q}$... these are the only *possible* rational zeros of the function.
- d) Graph the function to see which **are** the zeros of $f(x)$.
... and realize how nice your teacher is for not making you try ALL the numbers in part c.
- e) Using the graph from part d to get you started; find the zeros of $f(x)$ algebraically.
... (use synthetic division and/or factoring)