2.3 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE WITH MODELING

Learning Targets:

1. Be able to describe the end behavior of any polynomial using limit notation.
2. Be able to find the zeros of a polynomial by factoring.
3. Be able to find the zeros of a polynomial using your graphing calculator.
4. Understand how the multiplicity of a zero changes how the graph behaves when it hits the *x*-axis.
5. Use end behavior and multiplicity of zeros to sketch a polynomial by hand.
6. Use the regression capabilities of your calculator to model a cubic and quartic equation.

Our focus today is on polynomials of degree 3 (cubic), degree 4 (quartic), or higher.

When we looked at horizontal asymptotes, we talked about end behavior. End behavior is the behavior of the function (the y – values) as x approaches positive or negative infinity.

If you can remember the graphs of $y = x$, $y = -x$, $y = x^2$, and $y = -x^2$, then you can remember the end behavior of all polynomial functions. End behavior of any *polynomial* function can be described using the "leading coefficient" and highest power.

	Even Power	Odd Power
Positive Coefficient		
Negative Coefficient		

Example 1: Describe the end behavior of the polynomial function using $\lim\limits_{x\to\infty}$ and $\lim\limits_{x\to-\infty}$. Confirm graphically.

a) $f(x) = 3x^6 - 5x + 3$

b) $g(x) = -x^3 + 7x^2 - 8x + 9$

c) $h(x) = -2x^4 - 10x^2 + 87x - 1.07$

d) $k(x) = 7x^3 - 9$

<u>*Zeros of Polynomial Functions*</u>

Algebraic Analysis of Zeros: The *x*-intercepts of a function are also called the zeros. Factoring allows us to find these zeros algebraically. A polynomial of degree *n* has at most *n* zeros.

Example 2: Find the zeros of the function algebraically. Support graphically.

a) $f(x) = x^3 - 16x$ b) $g(x) = 3x^2 + x - 4$ c) $h(x) = 4x^3 - 14x^2 + 6x$

Graphical Analysis of Zeros

Example 3: Graph $f(x) = 2x^3 + 3x^2 - 7x - 6$ in a viewing window that shows all of its *x*-intercepts and find all of its zeros.

Example 4: List each zero and the power on each factor. Then graph each on your calculator. What do you notice?

a) $f(x) = x(x-3)^2$ b) $g(x) = -(x+2)^3(x-2)^4$ c) $h(x) = (x+1)(x-2)^2(x-4)^3$

Definition: Multiplicity of zeros:

The multiplicity of each zero is the number of times the factor occurs in the factored form of the polynomial.

At a zero with EVEN multiplicity, the graph of the function will ________________________________.

At a zero with ODD multiplicity, the graph of the function will __________________________________.

Example 5: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the *x*-axis at the corresponding *x*-intercept. Using what you know about end behavior and the zeros of the polynomial function, sketch the function.

a) $f(x) = x(x+2)^2$

b) $h(x) = -(x-2)^3(x+7)^2$

c) $g(x) = (x-2)^4(x+6)^2$

<u>Using Your Calculator's Regression Abilities to Fit Data to a Polynomial</u>

Example 6: Use cubic regression to fit a curve through the four points given in the table.

x	**3**	**4**	**6**	**8**
y	10	15	21	33

Example 7: Use quartic regression to fit a curve through the five points given in the table.

x	**0**	**4**	**5**	**7**	**13**
y	**-21**	**-19**	**-12**	**8**	**23**