Learning Targets:
1. Be able to describe the end behavior of any polynomial using limit notation.
2. Be able to find the zeros of a polynomial by factoring.
3. Be able to find the zeros of a polynomial using your graphing calculator.
4. Understand how the multiplicity of a zero changes how the graph behaves when it hits the x-axis.
5. Use end behavior and multiplicity of zeros to sketch a polynomial by hand.
6. Use the regression capabilities of your calculator to model a cubic and quartic equation.

Our focus today is on polynomials of degree 3 (cubic), degree 4 (quartic), or higher.

When we looked at horizontal asymptotes, we talked about end behavior. End behavior is the behavior of the function (the y-values) as x approaches positive or negative infinity.

If you can remember the graphs of \( y = x, y = -x, y = x^2, \) and \( y = -x^2, \) then you can remember the end behavior of all polynomial functions. End behavior of any polynomial function can be described using the “leading coefficient” and highest power.

<table>
<thead>
<tr>
<th>Even Power</th>
<th>Odd Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Coefficient</td>
<td></td>
</tr>
<tr>
<td>Negative Coefficient</td>
<td></td>
</tr>
</tbody>
</table>

Example 1: Describe the end behavior of the polynomial function using \( \lim_{x \to \infty} \) and \( \lim_{x \to -\infty}. \) Confirm graphically.

a) \( f(x) = 3x^6 - 5x + 3 \)

b) \( g(x) = -x^3 + 7x^2 - 8x + 9 \)

c) \( h(x) = -2x^4 - 10x^2 + 87x - 1.07 \)

d) \( k(x) = 7x^3 - 9 \)
Zeros of Polynomial Functions

Algebraic Analysis of Zeros: The x-intercepts of a function are also called the zeros. Factoring allows us to find these zeros algebraically. A polynomial of degree \( n \) has at most \( n \) zeros.

Example 2: Find the zeros of the function algebraically. Support graphically.

a) \( f(x) = x^3 - 16x \)

b) \( g(x) = 3x^2 + x - 4 \)

c) \( h(x) = 4x^3 - 14x^2 + 6x \)

Graphical Analysis of Zeros

Example 3: Graph \( f(x) = 2x^3 + 3x^2 - 7x - 6 \) in a viewing window that shows all of its x-intercepts and find all of its zeros.

Example 4: List each zero and the power on each factor. Then graph each on your calculator. What do you notice?

a) \( f(x) = x(x - 3)^2 \)

b) \( g(x) = -(x + 2)^3(x - 2)^4 \)

c) \( h(x) = (x + 1)(x - 2)^5(x - 4)^3 \)

Definition: Multiplicity of zeros:

The multiplicity of each zero is the number of times the factor occurs in the factored form of the polynomial.

At a zero with EVEN multiplicity, the graph of the function will ________________________________.

At a zero with ODD multiplicity, the graph of the function will ________________________________.
2.3 Polynomial Functions of Higher Degree with Modeling

Example 5: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Using what you know about end behavior and the zeros of the polynomial function, sketch the function.

a) \( f(x) = x(x + 2)^3 \)
b) \( h(x) = -(x - 2)^3(x + 7)^2 \)
c) \( g(x) = (x - 2)^4(x + 6)^2 \)

Using Your Calculator’s Regression Abilities to Fit Data to a Polynomial

Example 6: Use cubic regression to fit a curve through the four points given in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>33</td>
</tr>
</tbody>
</table>

Example 7: Use quartic regression to fit a curve through the five points given in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-21</td>
<td>-19</td>
<td>-12</td>
<td>8</td>
<td>23</td>
</tr>
</tbody>
</table>