2.3 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE WITH MODELING

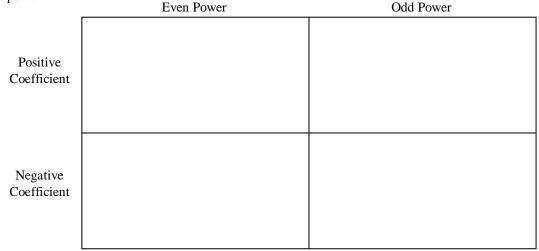
Learning Targets:

- 1. Be able to describe the end behavior of any polynomial using limit notation.
- 2. Be able to find the zeros of a polynomial by factoring.
- 3. Be able to find the zeros of a polynomial using your graphing calculator.
- 4. Understand how the multiplicity of a zero changes how the graph behaves when it hits the x-axis.
- 5. Use end behavior and multiplicity of zeros to sketch a polynomial by hand.
- 6. Use the regression capabilities of your calculator to model a cubic and quartic equation.

Our focus today is on polynomials of degree 3 (cubic), degree 4 (quartic), or higher.

When we looked at horizontal asymptotes, we talked about end behavior. End behavior is the behavior of the function (the y – values) as x approaches positive or negative infinity.

If you can remember the graphs of y = x, y = -x, $y = x^2$, and $y = -x^2$, then you can remember the end behavior of all polynomial functions. End behavior of any *polynomial* function can be described using the "leading coefficient" and highest power.



Example 1: Describe the end behavior of the polynomial function using lim and lim . Confirm graphically.

a) $f(x) = 3x^6 - 5x + 3$ b) $g(x) = -x^3 + 7x^2 - 8x + 9$

c)
$$h(x) = -2x^4 - 10x^2 + 87x - 1.07$$

d) $k(x) = 7x^3 - 9$

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Zeros of Polynomial Functions

Algebraic Analysis of Zeros: The *x*-intercepts of a function are also called the zeros. Factoring allows us to find these zeros algebraically. A polynomial of degree *n* has at most *n* zeros.

Example 2: Find the zeros of the function algebraically. Support graphically.

a)
$$f(x) = x^3 - 16x$$

b) $g(x) = 3x^2 + x - 4$
c) $h(x) = 4x^3 - 14x^2 + 6x$

Graphical Analysis of Zeros

Example 3: Graph $f(x) = 2x^3 + 3x^2 - 7x - 6$ in a viewing window that shows all of its *x*-intercepts and find all of its zeros.

Example 4: List each zero and the power on each factor. Then graph each on your calculator. What do you notice?

a) $f(x) = x(x-3)^2$ b) $g(x) = -(x+2)^3(x-2)^4$ c) $h(x) = (x+1)(x-2)^2(x-4)^3$

Definition: Multiplicity of zeros:
The multiplicity of each zero is the number of times the factor occurs in the factored form of the polynomial.
At a zero with EVEN multiplicity, the graph of the function will
At a zero with ODD multiplicity, the graph of the function will

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Example 5: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the *x*-axis at the corresponding *x*-intercept. Using what you know about end behavior and the zeros of the polynomial function, sketch the function.

a)
$$f(x) = x(x+2)^2$$

b) $h(x) = -(x-2)^3(x+7)^2$
c) $g(x) = (x-2)^4(x+6)^2$

Using Your Calculator's Regression Abilities to Fit Data to a Polynomial

Example 6: Use cubic regression to fit a curve through the four points given in the table.

x	3	4	6	8
у	10	15	21	33

Example 7: Use quartic regression to fit a curve through the five points given in the table.

x	0	4	5	7	13
у	-21	-19	-12	8	23