2.3 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE WITH MODELING

Our focus today is on polynomials of degree 3 (cubic), degree 4 (quartic), or higher.

When we looked at horizontal asymptotes, we talked about end behavior. End behavior is the behavior of the function (the y – values) as x approaches positive or negative infinity.

Example: Sketch each of the following equations and describe the end behavior.

a) $y = 3x^2$ b) $y = -3x^2$ c) $y = 2x^3$ d) $y = -2x^3$

End behavior of any *polynomial* function can be described using the box below.

Example: Describe the end behavior of the polynomial function using $\lim\limits_{x\to\infty}$ and $\lim\limits_{x\to-\infty}$. Confirm graphically.

a) $f(x) = 3x^6 - 5x + 3$

b) $g(x) = -x^3 + 7x^2 - 8x + 9$

c) $h(x) = -2x^4 - 10x^2 + 87x - 1.07$

d) $k(x) = 7x^3 - 9$

Zeros of Polynomial Functions

Algebraic Analysis of Zeros

Once factored, we can use the zero product property to find the zeros of a polynomial algebraically.

Example: Find the zeros of the function algebraically. Support graphically.

a) $f(x) = x^3 - 16x$

b) $g(x) = 3x^2 + x - 4$

c) $h(x) = 4x^3 - 14x^2 + 6x$

Graphical Analysis of Zeros

Example: Graph the function in a viewing window that shows all of its *x*-intercepts and find all of its zeros.

$$f(x) = 2x^3 + 3x^2 - 7x - 6$$

Example: Graph each of the functions below. What relationship between the zeros and the powers do you notice?

a) $f(x) = x(x-3)^2$

b) $g(x) = -(x+2)^3(x-2)^4$

c) $h(x) = (x+1)(x-2)^2(x-4)^3$

Definition: Multiplicity

Example: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the *x*-axis at the corresponding *x*-intercept. Using what you know about end behavior and the zeros of the polynomial function, sketch the function.

a) $f(x) = x(x+2)^2$

b) $h(x) = -(x-2)^3(x+7)^2$

c) $g(x) = (x-2)^4(x+6)^2$

Using Your Calculator to Fit Data to a Polynomial

Example: Use cubic regression to fit a curve through the four points given in the table.

x	**3**	**4**	**6**	**8**
y	10	15	21	33

Example: Use quartic regression to fit a curve through the five points given in the table.

x	**0**	**4**	**5**	**7**	**13**
y	**-21**	**-19**	**-12**	**8**	**23**