2.1 LINEAR AND QUADRATIC FUNCTIONS AND MODELING

Learning Targets:
1. Understand what a polynomial function looks like.
2. Understand that Average Rate of Change implies slope between two points.
3. Model Linear functions using function notation and the regression capabilities of your calculator.
4. Find the vertex and graph a quadratic function in standard, intercept, and vertex forms.
5. Model Quadratic functions in vertex form.
6. Use the projectile motion model to find the highest point a projectile reaches, and when it reaches that point.

Chapter 2 deals with polynomial functions and you will learn about various aspects of these types of functions. Our first goal is to define polynomial functions and then to identify which functions are polynomials.

Definition of a Polynomial Function

A function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$,

where n = ________________________________ and n is an ____________________________.

Example 1: Which of the following functions are polynomial functions? For those that are, state the degree and leading coefficient. For those that are not polynomials state why not.

$f(x) = 3x - 5x^2$ $h(x) = \sqrt{x^2 - 4}$ $s(t) = 2t^4 + 5t^3$ $g(x) = 6$ $k(x) = 4x^{\frac{2}{3}} - 5$

For the remainder of this section we will deal only with functions whose degree is less than or equal to 2.

Polynomials of degree 0 are called ____________________.

Polynomials of degree 1 are called ____________________.

Polynomials of degree 2 are called ____________________.

Modeling Linear Functions

Example 2: Using function notation, if $f(a) = b$, then the function contains the point ______________ .

Example 3: The **average rate of change** of a function between two points $(a, f(a))$ and $(b, f(b))$ is given by

Example 4: Write the equation of the linear function, f, if you know $f(2) = -7$ and $f(-1) = 5$.

Example 5: The table below shows the relationship between the number of Calories and the number of grams of fat in 9 different hamburgers from various fast-food restaurants.

Calories	720	530	510	500	305	410	440	320	598
Fat (g)	46	30	27	26	13	20	25	13	26

a) Find the linear regression model for this data.

b) In the context of this problem, what does the slope mean?

Modeling Quadratic Functions

Quadratic Functions are going to be given to you in the following forms:

$f(x) = ax^2 + bx + c$ $\qquad\qquad$ $f(x) = a(x-h)^2 + k$

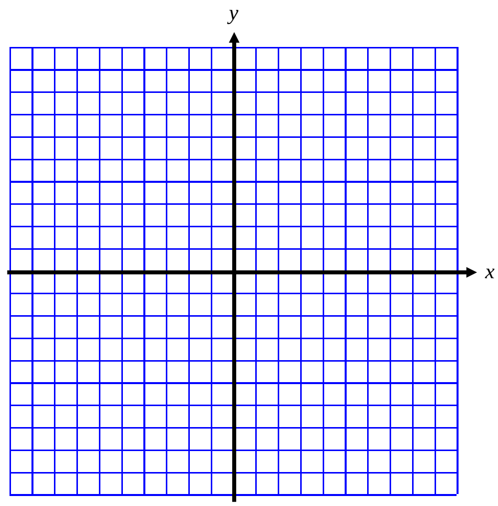
You need to be able to graph each form. The most important point will be the vertex.

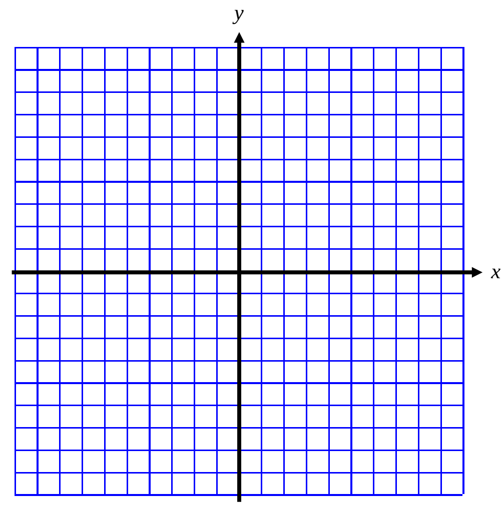
Example 6: Find the vertex of the quadratic function $f(x) = 2x^2 - 6x + 11$.

Example 7: Find the vertex of the quadratic function $f(x) = 3(x+5)^2 - 2$.

Once you find the vertex of a parabola, you can graph the remaining points by following the pattern below

Example 8: Graph the two quadratic functions from the previous examples.





Example 9: Find the equation of a quadratic function of the form $f(x) = a(x-h)^2 + k$ that passes through (–4, 15) and has it's vertex at (–2, 3).

Example 10: Change the quadratic function you wrote in the last example into the form $f(x) = ax^2 + bx + c$.

Ignoring air resistance, the height, *s*, of a free-falling object after *t* seconds can be modeled by the quadratic function

$s(t) = -16t^2 + v_0 t + s_0$, if *s* is in feet … or … $s(t) = -4.9t^2 + v_0 t + s_0$, if *s* is in in meters

Where v_0 = ____, and

s_0 = ____.

Example 11: At the Bakersville Fourth of July celebration, fireworks are shot by remote control into the air from a pit that is 10 feet below the earth's surface.

a) Find an equation that models the height of an aerial bomb *t* seconds after it is shot upwards with an initial velocity of 80 ft/sec.

b) Find the vertex of the quadratic function.

c) What is the maximum height above ground level that the aerial bomb will reach?

d) How many seconds after it was launched will it take to reach that height?

Example 12: The Welcome Home apartment rental company has 1600 units available, of which 800 are currently rented at \$300 per month. A market survey indicates that each \$5 decrease in monthly rent will result in 20 new leases.

a) Determine a function $R(x)$ that models the total rental income realized by Welcome Home, where *x* is the number of \$5 decreases in monthly rent.

b) Find a graph of $R(x)$ for the rent levels between \$175 and \$300 (that is $0 \leq x \leq 25$) that clearly shows a maximum for $R(x)$. What rent will yield Welcome Home the maximum monthly income?