1.4 BUILDING FUNCTIONS FROM FUNCTIONS

Learning Target for 1.4

1. Be able to Add, Subtract, Multiply, and Divide two functions
2. Be able to Compose two functions
3. Find an Inverse Functions Graphically, Numerically, and Algebraically
4. Prove two functions are Inverses

Creating functions from other functions can be done in a variety of ways. We are going to add, subtract, multiply, divide, and compose functions to create new functions.

Example 1: Consider the two functions $f(x) = 6 - x$ and $g(x) = \sqrt{x+2}$. What is the Domain of each?

a) Find $(f+g)(x)$. State the Domain of the new function.

b) Find $(f-g)(x)$. State the Domain of the new function.

c) Find $(fg)(x)$. State the Domain of the new function.

d) Find $\left(\frac{f}{g}\right)(x)$. State the Domain of the new function.

Composite Functions

When the range of one function is used as the domain of a second function we call the entire function a composite function.

We use the notation $(f \circ g)(x) = f(g(x))$ to describe composite functions.

This is read as "*f* composed with *g*" or "*f* of *g* of *x*".

The function *f* in the example above is an example of a composite function. The linear function "$x + 2$" is applied first, then the square root function.

Example 2: Suppose $f(x) = 1 - x^2$ and $g(x) = \sqrt{x}$.

a) Find $g(f(x))$. What is the domain of $g(f(x))$?

b) Find $f(g(x))$. What is the domain of $g(f(x))$?

Example 3: Suppose $f(x)=\sqrt{x-5}$, and $g(x)=x^2+5$.

a) Find $f(g(x))$.

b) Find $g(f(x))$.

<u>*Inverse Functions*</u>

A function has an inverse function if and only if the original function passes the Horizontal Line Test. The Horizontal Line Test works just like the Vertical Line Test (it's just horizontal ☺). All an inverse function does is switch the *x* and *y* or the domain and range.

Example: Does $y=x^2+5x$ have an inverse? Why or why not?

Example: Does $y=x^3+x$ have an inverse? Why or why not?

Once we know whether a function has an inverse, our next task is to find an equation and/or a graph for the inverse.

Finding the Inverse Graphically

Reflect the graph of the original function over the line $y = x$.

Finding the Inverse Numerically

Plot the reverse of the coordinates.

Finding the Inverse Algebraically

Switch the *x* and *y* in the original equation, then solve the new equation for *y* in order to write *y* as a function of *x*. Your NEW *y* can be written as $f^{-1}(x)$

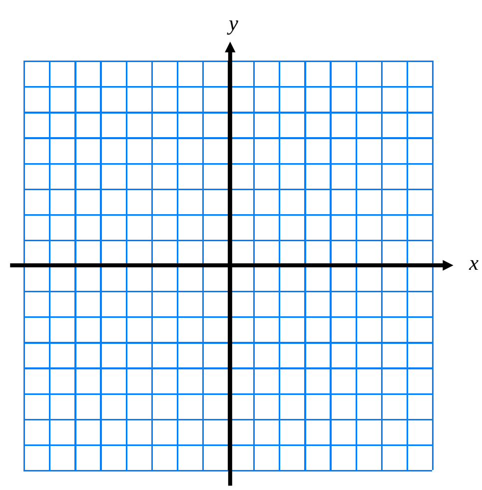
Example 4: Let $f(x)=x^3-1$.

a) Graph the function on the grid to the right.

b) Draw the line $y = x$

c) Reflect the graph of $f(x)$ over the line $y = x$.

d) Find the inverse of the function algebraically.



Verifying Inverses

It is one thing to find the inverse function (either graphically or algebraically), but it is another to verify that two functions are actually inverses. Whenever you are verifying anything in mathematics, you must go back and use the definition.

Definition: Inverse Function

A function $f(x)$ has an inverse $f^{-1}(x)$ if and only if $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$

Example 5: According to this definition, how many composite functions must you use to check whether or not two functions are inverses of each other?

Example 6: **Find** $f^{-1}(x)$ and **verify** if $f(x) = 3x - 2$.

Example 7: Find $f^{-1}(x)$ and verify if $f(x) = \dfrac{x+3}{x-2}$.