

1.4 BUILDING FUNCTIONS FROM FUNCTIONS

Creating functions from other functions can be done in a variety of ways. We are going to add, subtract, multiply, divide, and compose functions to create new functions.

Example: Consider the two functions $f(x) = \sqrt{x+2}$ and $g(x) = 6-x$. What is the Domain and Range of each?

a) Find $(f+g)(x)$. State the Domain and Range of the new function.

b) Find $(f-g)(x)$. State the Domain and Range of the new function.

c) Find $(fg)(x)$. State the Domain and Range of the new function.

d) Find $\left(\frac{f}{g}\right)(x)$. State the Domain and Range of the new function.

Composite Functions

When the range of one function is used as the domain of a second function we call the entire function a composite function.

We use the notation $(f \circ g)(x) = f(g(x))$ to describe composite functions.

This is read as "f composed with g" or "f of g of x".

The function f in the example above is an example of a composite function. The linear function " $x+2$ " is applied first, then the square root function.

Example: Suppose $f(x) = 1-x^2$ and $g(x) = \sqrt{x}$.

a) Find $g(f(x))$. What is the domain and range of $g(f(x))$?

b) Find $f(g(x))$. What is the domain and range of $f(g(x))$?

Example: Suppose $f(x) = \frac{2x-1}{x+3}$, and $g(x) = \frac{3x+1}{2-x}$.

a) Find $f(g(x))$.

b) Find $g(f(x))$.

Inverse Functions

In technical jargon, an inverse of a function maps the elements of the range to the elements of the domain. In English, this means that the inverse of a function reverses the domain and range. Not all graphs were defined as functions, and we had the *vertical line test* to determine whether a graph was or was not a function. Similarly, not all functions have an inverse, and we have the *horizontal line test* to determine whether or not a function has an inverse.

Definition: One – to – One Function

A function $f(x)$ is **one – to – one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

A function that is one – to – one has an inverse.

The definition above can be seen graphically with the use of a horizontal line test. If there are two x – values for any given y – value of function, then the function does NOT have an inverse.

Example: Does $y = x^2 + 5x$ have an inverse? Why or why not?

Example: Does $y = x^3 + x$ have an inverse? Why or why not?

Once we know whether a function has an inverse, our next task is to find an equation and/or a graph for the inverse.

Finding the Inverse Graphically

Reflect the graph of the original function over the line $y = x$.

Finding the Inverse Numerically

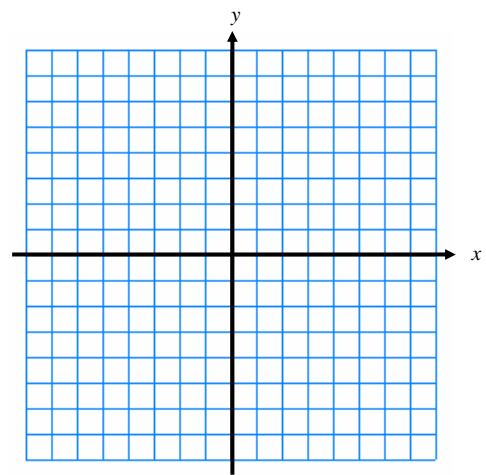
Plot the reverse of the coordinates.

Finding the Inverse Algebraically

Switch the x and y in the original equation, then solve the new equation for y in order to write y as a function of x .

Example: Let $f(x) = x^3 - 1$.

- Graph the function on the grid to the right.
- Draw the line $y = x$
- Reflect the graph of $f(x)$ over the line $y = x$.
- Find the inverse of the function algebraically.



- Use your graphing calculator to verify your answer to part *d*.

Verifying Inverses

It is one thing to find the inverse function (either graphically or algebraically), but it is another to verify that two functions are actually inverses. Whenever you are verifying anything in mathematics, you must go back and use the definition.

Definition: Inverse Function

A function $f(x)$ has an inverse $f^{-1}(x)$ if and only if $f(f^{-1}(x)) = x = f^{-1}(f(x))$

Example: According to this definition, how many composite functions must you use to check whether or not two functions are inverses of each other?

Example: Find $f^{-1}(x)$ and verify if $f(x) = 3x - 2$.

Example: Find $f^{-1}(x)$ and verify if $f(x) = \frac{x+3}{x-2}$.