

1.2 FUNCTIONS AND THEIR PROPERTIES

Learning Targets for 1.2

1. Determine whether a set of numbers or a graph is a function
2. Find the domain of a function given an set of numbers, an equation, or a graph
3. Describe the type of discontinuity in a graph as removable or non-removable
4. For a given function, describe the intervals of increasing and decreasing
5. Label a graph as bounded above, bounded below, bounded, or unbounded.
6. Find all relative extrema on a graph
7. Understand the difference between absolute and relative extrema
8. Describe the symmetry of a graph as odd or even
9. Prove a function is odd, even, or neither
10. Given a rational equation, find vertical asymptotes and holes (if they exist)
11. Given a rational equation, find the end behavior model
12. Given a rational equation, describe the end behavior using end behavior and limit notation.
13. Given a rational equation, use the end behavior to find any horizontal asymptotes.
14. Given a rational equation, determine when slant (oblique) asymptotes exist and find them.

This section is full of vocabulary (obvious from the list above?). We will be investigating functions, and you will need to answer questions to determine how each of these properties are applied to various functions.

Function Definition and Notation

In the last chapter, we used the phrase “ y is a function of x ”. But what is a function? In Algebra 1, we defined a function as a rule that assigned one and only one (a unique) output for every input. We called the input the *domain* and the output the *range*. Usually, the set of possible x -values is the domain, and the resulting set of possible y -values is the range.

Definition: Function

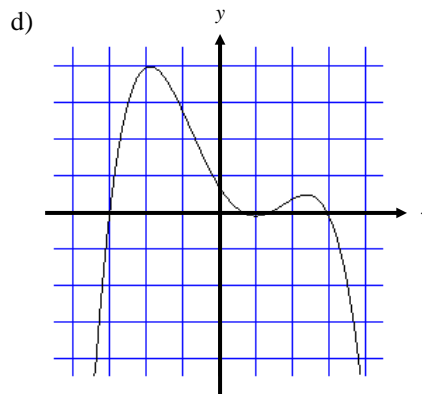
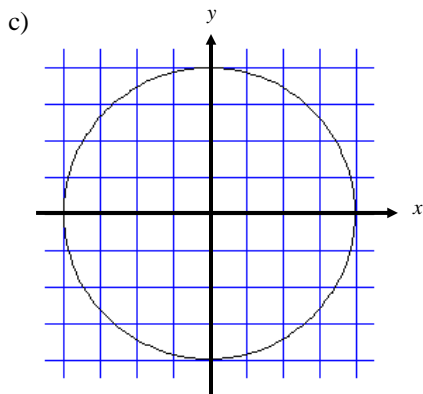
A **function** from a set D to a set R is a rule that assigns a unique element in R to each element in D .

To determine whether or not a graph is a function, you can use the **vertical line test**. If any vertical line intersects a graph more than once, then that graph is NOT a function.

Example 1: A relation is ANY set of ordered pairs. State the domain and range of each relation, then tell whether or not the relation is a function.

a) $\{(-3,0), (4,2), (2,-6)\}$

b) $\{(4,-2), (4,2), (9,-3), (-9,-3)\}$



For many functions, the domain is all real numbers, or \mathbb{R} . We typically start with this, and then see if there are any values of x that cannot be used.

The domain of a function can be restricted for 3 reasons that you need to be aware of in this course:

1. $\sqrt{\text{NO negatives}}$... no negative numbers inside a square root
2. $\frac{**}{\text{NO zeros}}$... no zeros in the denominator of a fraction
3. $\log(\text{NO negatives ... and NO zeros})$ no negatives and no zeros inside a logarithm

Example 2: Without using a graphing calculator, what is the domain of each of the following functions?

a) $y = \sqrt{8-x}$

c) $y = \ln x$.

b) $h(x) = \frac{x^2-9}{x+3}$

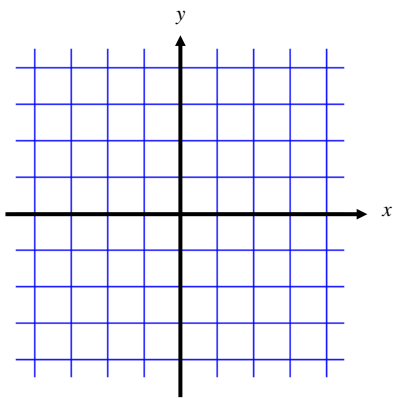
d) $y = \frac{\sqrt{x+2}}{x^2-x-12}$

Continuity

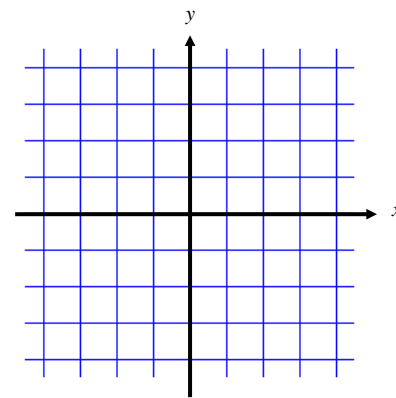
In non – technical terms, a function is continuous if you can draw the function “without ever lifting your pencil”.

Example 3: Graph each of the following functions. What do you notice? What happens when $x = 2$ on the graph of b ?

a) $f(x) = x + 2$

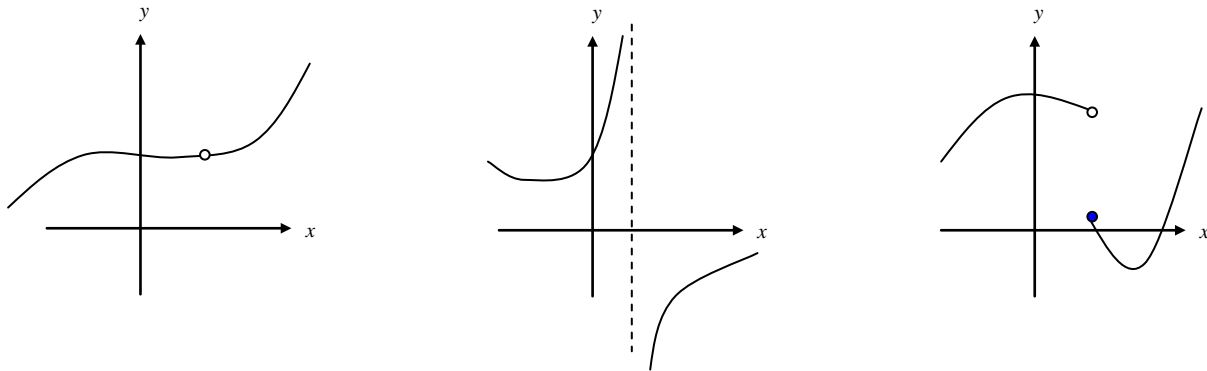


b) $g(x) = \frac{x^2-4}{x-2}$

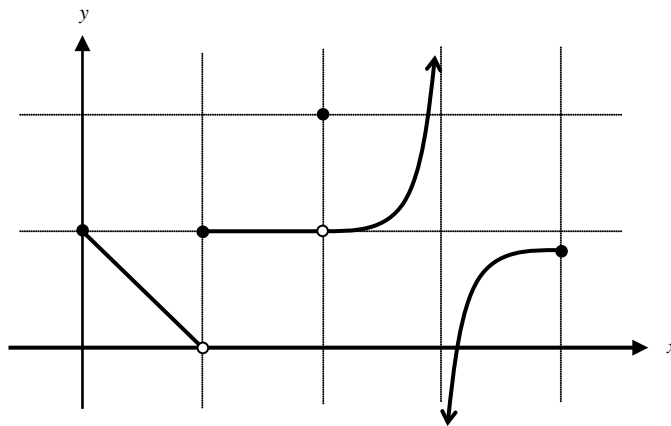


Example 4: What is the domain and range of the two functions above?

Example 5: The following graphs demonstrate three types of discontinuous graphs. Label each type as removable or non-removable.



Example 6: Tell where the function graphed below is discontinuous. Describe each discontinuity.



Example 7: What is the domain and range of the function above?

Example 8: Using your calculator, graph each of the following functions. Determine if it has discontinuity at $x = 0$. If there is a discontinuity, tell whether it is removable or non-removable.

a) $f(x) = x^3 - 2x^2 + 1$

b) $g(x) = \frac{9}{x}$

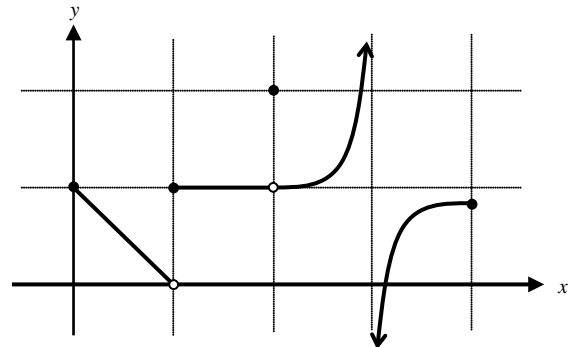
c) $h(x) = \frac{x}{x-3}$

d) $k(x) = \frac{x^2 - 2x}{x}$

Increasing/Decreasing Functions

While there are technical definitions for increasing and decreasing, just remember to read the graph LEFT to RIGHT.

Example 9: Using the graph below, what intervals is the function increasing? Decreasing? Constant?

Boundedness

You need to understand the difference between the following terms:

BOUNDED BELOW:

BOUNDED ABOVE:

BOUNDED:

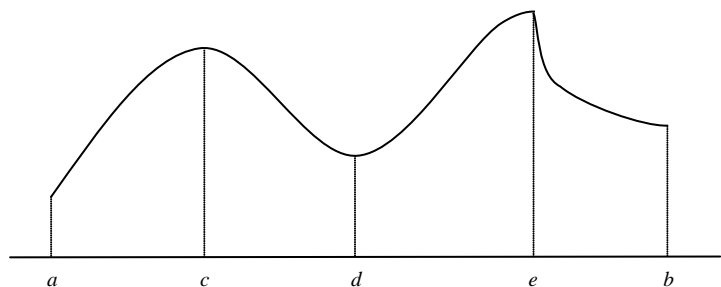
Local and Absolute Extrema

Extrema is the plural form of one extreme value. Extrema is one word that includes maximums and minimums.

Local Extrema are points bigger (or smaller) than _____.

Absolute Extrema are points bigger (or smaller) than _____.

Example: Suppose the following function is defined on the interval $[a, b]$. Identify the location and types of extrema.



Example 10: Use your graphing calculator, to graph the function $g(x) = -x^3 + 2x - 3$.

- Identify all extrema.
- Identify the intervals on which the function is increasing, decreasing, or constant.

Symmetry

The next topic we concern ourselves with when dealing with functions is the idea of symmetry. Symmetry on a graph means the functions “look the same” on one side as it does on another. We are most concerned with the types of symmetry that can be explored numerically and algebraically in terms of ODD and EVEN functions.

3 Types of Symmetry

Symmetry with respect to the y-axis:

EVEN FUNCTIONS

Graphically

Numerically

Algebraically

Symmetry with respect to the origin:

ODD FUNCTIONS

Graphically

Numerically

Algebraically

Symmetry with respect to the x-axis:

NOT A FUNCTION

Graphically

Numerically

Algebraically

Odd vs Even Functions

While there are many functions out there that are neither even nor odd, your concern with odd and even functions is twofold ...

#1: Identify graphs of functions that are Odd or Even

#2: PROVE a function is Odd or Even

While the first item above can be done graphically, numerically, or algebraically, the second is done ONLY algebraically.

Example 11: Prove each function is odd, even, or neither.

a) $f(x) = 5x^2 + 5$

b) $g(x) = x^2 + 8x + 12$

c) $h(x) = -4x^3 + 2x$

Vertical Asymptotes

Example 12: Graph the function $f(x) = \frac{1}{x-1}$. What happens at $x = 1$? ... Why?

To describe a vertical asymptote, we must talk about what happens to the y -values of a function as the x -values get “close” to a certain number.

We use the notation $x \rightarrow 1$ to say _____.

If we only want x to approach 1 from the right side, we use the notation _____.

If we only want x to approach 1 from the left side, we use the notation _____.

♫: In calculus, we use what is called “limit notation” ... $\lim_{x \rightarrow c^+} f(x) = \pm\infty$ or $\lim_{x \rightarrow c^-} f(x) = \pm\infty$.

Vertical asymptotes occur in rational functions when _____.

If you plug a point into your function and get _____, you should look for a removable discontinuity ... a.k.a. “a hole”.

Horizontal Asymptotes and End Behavior

Horizontal Asymptotes occur when the function values (y -values) get “close” to a specific number as the x -values get really, really large in the positive direction (∞) or negative direction ($-\infty$).

As $x \rightarrow \pm\infty$, we say the *end behavior* of the function is a description of what $f(x)$ approaches.

IF $f(x) \rightarrow$ "a number" as $x \rightarrow \pm\infty$, we say the function has a *horizontal asymptote*.

Example 13: Graph the function $g(x) = \frac{4x^3}{27-x^3}$. Where does the horizontal asymptote occur?

The end behavior model of a polynomial is the leading coefficient and the highest power of the variable.

A rational function is just two polynomials divided, so the end behavior model of a rational function is just the end behavior model of the numerator divided by the end behavior model of the denominator.

Example 14: What is the end behavior model for $g(x) = \frac{4x^3}{27-x^3}$? How is this related to the horizontal asymptote.

Example 15: Consider the function $h(x) = \frac{1}{x}$. As $x \rightarrow \infty$, where does $h(x)$ approach?

Summary for Horizontal Asymptotes

For rational functions we have the following results.

If $f(x) = ax^m + \dots$ and $g(x) = bx^n + \dots$ then $\frac{f(x)}{g(x)}$ takes on three different forms.

	End Behavior Model	End Behavior	Asymptote
$m = n$			
$m < n$			
$m > n$			

Example 16: Find the end behavior models of each function and determine what (if any) the horizontal asymptotes are.

a) $f(x) = \frac{5x^5 - 7x^3 + 2x - 8}{10x^5 + 16x^4 - 3x^2 + 9}$

b) $g(x) = \frac{5x^8 - 7x^3 + 2x - 8}{10x^5 + 16x^4 - 3x^2 + 9}$

c) $h(x) = \frac{5x^4 - 7x^3 + 2x - 8}{10x^6 + 16x^4 - 3x^2 + 9}$

Example 17: Go back and find the slanted (or oblique) asymptote for the graph in part *b* above.

Example 18: Find ALL vertical and horizontal asymptotes for the following functions.

a) $f(x) = \frac{2x-1}{3x+5}$

b) $g(x) = \frac{(3x-5)(x-8)}{x^2-4}$

c) $h(x) = \frac{2x-9}{x^2-x-6}$

d) $k(x) = \frac{2x}{x^2-x-6}$