Logic Lesson 2

Definition: Two statements are ***Logically Equivalent*** if the corresponding truth values for each of the possible combinations of true values are the same.

Let's look at the truth table for a statement, the converse of a statement, and the conjunction joining them both.

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$p \leftrightarrow q$
T
F
F
T

The statement $p \leftrightarrow q$ is called a ***Biconditional Sentence***, or equivalence. We can write an equivalence in many ways. Here are a few examples…

- if p then q and if q then p … "$(p \to q) \wedge (q \to p)$"
- if p then q, and conversely
- p if and only if q
- p iff q (*iff* is an acceptable abbreviation for *if and only if.*)

Note: A definition is always an equivalence.

Contrapositive:

If you are given the conditional sentence "If *p*, then *q*" , the new conditional sentence "If ~*q*, then ~*p*" is called the contrapositive of the original statement. Look at their truth tables. What do you notice?

Complete the truth table…

p	q	$p \to q$	$\sim q \to \sim p$
T	T		
T	F		
F	T		
F	F		

Since $p \to q$ and $\sim q \to \sim p$ have the same values in the truth table, they are logically equivalent. This is useful in mathematics when trying to prove something indirectly. Instead of proving "*p implies q*", you can show that "*not q implies not p*".