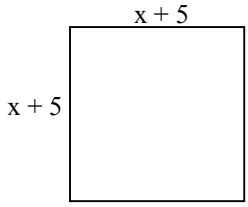
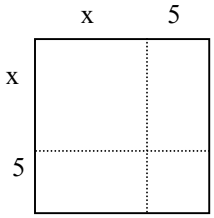
Completing the Square

Review… how do you solve $(x+5)^2 = 20$?

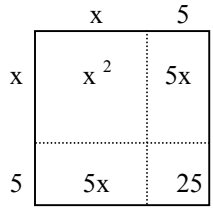
Left side of this equation is a perfect square…
Geometrically this is as follows,



Break it up like this →

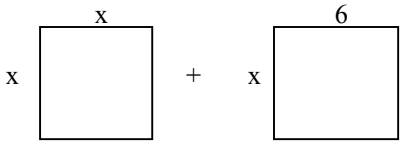


Look at the areas →

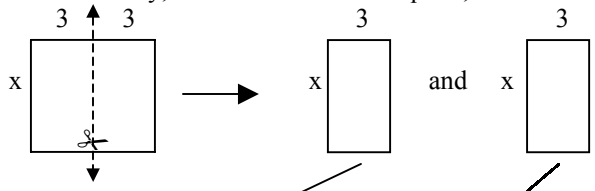


So, $(x+5)^2 = x^2 + 10x + 25$!

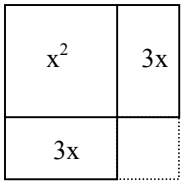
Now, let's take a look at $x^2 + 6x$ geometrically.



Geometrically, to make $x^2 + 6x$ a square, we need to cut the 6x piece in half and separate as follows:



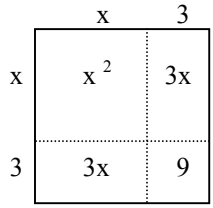
Slide one of the 3x pieces against the side of the x^2 piece. Rotate the other and slide against the bottom of the x^2 piece like this…



Notice the missing area? How much is it? _____

So, in order to *complete the square* you must add $\underline{9}$ to $x^2 + 6$.

This means that $x^2 + 6x + 9 = (x+3)^2$, or geometrically we have,



Algebraically, how do we accomplish completing the square? First notice, the coefficient of x^2 needs to be one (1). If it is not, you must first divide through to make the coefficient equal to 1. Then…

1. Take ½ the coefficient of x
2. Square the value from step 1 and add this to the original expression.
3. The new expression now factors to a perfect square (x + d) where d is the value obtained in step 1.

Ex. Complete the square:

$$x^2 + 2x + \_\_\_$$

$$\frac{1}{2}(2) = 1$$

$$1^2 = 1$$

$$x^2 + 2x + \underline{1}$$

$$(x+1)^2$$

$$x^2 + 3x + \_\_\_$$

$$\frac{1}{2}(3) = \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$x^2 + x + \underline{\frac{9}{4}}$$

$$\left(x + \frac{3}{2}\right)^2$$

You can use this concept to solve equations as well.

Consider $2x^2 - 6x + 3 = 0$. Solve for x by completing the square.

First, make the coefficient of x^2 equal to 1, by dividing all the terms by 2 to obtain,

$$x^2 - 3x + \tfrac{3}{2} = 0$$

Next, get all the x terms on the left – side and the constant terms on the right.

$$x^2 - 3x = -\tfrac{3}{2}$$

Thirdly, complete the square on the left.

$$x^2 - 3x + \_\_\_ = -\tfrac{3}{2} + \_\_\_$$

$$\frac{1}{2}(-3) = -\frac{3}{2}$$

$$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

This means in order to complete the square, you need to add $\frac{9}{4}$ to my left side of the equation. You also need to add $\frac{9}{4}$ to the other side of the equation. Why? You should then have,

$$x^2 - 3x + \tfrac{9}{4} = -\tfrac{3}{2} + \tfrac{9}{4}$$

Factoring the left side, and simplifying the right gives you,

$$\left(x + \tfrac{3}{2}\right)^2 = \tfrac{3}{4}$$

You should now solve like you did the review problem on the other side of the page in order to verify the solutions given below. The solutions are

$$x = \frac{3 \pm \sqrt{3}}{2} \approx 2.37 \quad \text{or} \quad \approx 0.63$$