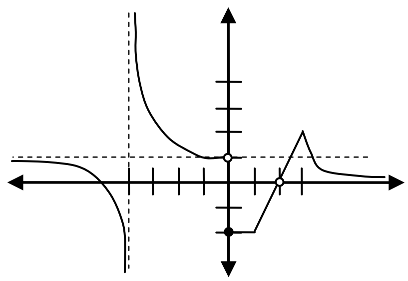
MAT 125
Survey of Calculus

Final Review Problems

1. Use the graph of $f(x)$ below to find the following:



(a) $\lim_{x\to 0^+} f(x)$
(b) $\lim_{x\to -4^+} f(x)$
(c) $\lim_{x\to 2} f(x)$
(d) $\lim_{x\to 3} f(x)$
(e) $\lim_{x\to \infty} f(x)$
(f) $\lim_{x\to -\infty} f(x)$
(g) points where f is not continuous
(h) points where f is not differentiable
(i) intervals where $f' > 0$
(j) intervals where $f' < 0$
(k) intervals where $f'' > 0$
(l) intervals where $f'' < 0$

2. Find the following limits:

(a) $\lim_{x\to -1} \dfrac{x^2+x}{x^2-x-2}$ (b) $\lim_{x\to 2} \dfrac{x^2+x}{x^2-x-2}$ (c) $\lim_{x\to \infty} \dfrac{x^2+x}{x^2-x-2}$

3. Find $f'(x)$ using the *definition* of the derivative if $f(x) = 3x^2 - 1$.

4. Find $f'(x)$ using the rules of derivatives:
(a) $f(x) = (x+2)^3(x-3)^5$
(b) $f(x) = \sqrt{3x^2+1}$
(c) $f(x) = \dfrac{e^{2x}}{6x}$
(d) $f(x) = 3^x - x^{1/3}$
(e) $f(x) = \ln(\sqrt{x+1})$
(f) $f(x) = 5\ln(x^2 - x + 3)$
(g) $f(x) = 3\log_2(x) - 1$

5. The circular area *A*, in square centimeters, of a healing wound is given by $A(r) = \pi r^2$ where *r* is the radius, in centimeters.
(a) Find the rate of change of the area with respect to the radius.
(b) Find the rate of change of area when $r = 0.2$ cm.
(c) Interpret your answer to part (b).

6. Let $f(x) = \dfrac{x^2}{x^2-4}$.

(a) Find the equations of any horizontal or vertical asymptotes.
(b) Find the critical numbers of the function.
(c) Use the First Derivative Test to classify the critical numbers as relative maximum, relative minimum or neither.
(d) Find the intervals where *f* is concave up and where *f* is concave down.
(e) Find the coordinates of any inflection points of *f*.

7. Find all relative extrema of $y = e^{x^2} + 1$

8. After an injection, the amount of a medication *A* in the bloodstream decreases after time *t*, in hours. Suppose that under certain conditions *A* is given by

$$A(t) = \frac{A_0}{t^2 + 1},$$

where A_0 is the initial amount of the medication given. Assume that an initial amount of 100 cc is injected.

(a) Find $\lim_{t \to \infty} A(t)$.

(b) Find the maximum value of the injection over the interval $[0, \infty)$.

(c) According to this function, does the medication ever completely leave the bloodstream? Explain your answer.

9. A 300 room hotel in Las Vegas is filled to capacity every night at $80 a room. For each $1 increase in rent, 3 fewer rooms are rented. If each rented room costs $10 to service per day, how much should the management charge for each room to maximize gross profit? What is the maximum gross profit? (*Hint*: let *x* represent the number of $1 increases in room price).

10. A model for advertising response is given by

$$N(a) = 1000 + 200 \ln a, \quad a \geq 1$$

where $N(a)$ is the number of units sold and *a* is the amount spent on advertising, in thousands of dollars.

(a) How many units were sold after spending $1000 ($a$ = 1) on advertising?

(b) Find $N'(a)$ and $N'(10)$.

(c) Discuss $\lim_{a \to \infty} N'(a)$. Does it make sense to spend more and more dollars on advertising?

11. The population of France in July 2001 was estimated to be 60 million. At that time it was estimated that the population was growing exponentially at a rate of 0.37% per year; that is,

$$\frac{dp}{dt} = 0.0037P,$$

where *t* is the time, in years.

(a) Find the function that satisfies the equation. Assume that $P_0 = 60$ and $k = 0.0037$.

(b) Estimate the population of France in 2010.

(c) After what period of time will the population double that in 2001?

12. Evaluate the following indefinite integrals:

(a) $\int x^3 - x + 2 \quad dx$

(b) $\int (x+2)^2 \quad dx$

(c) $\int \sqrt{x-1} \quad dx$

(d) $\int e^{5x} - 5 \quad dx$

(e) $\int \frac{1}{x} - x^a \quad dx$

(f) $\int \frac{13}{\sqrt[4]{x^3}} dx$

13. Evaluate the following definite integrals:

(a) $\int_0^9 \sqrt{x} \quad dx$

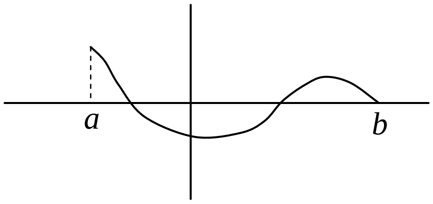
(b) $\int_1^4 \frac{2}{x} dx$

(c) $\int_{-2}^3 7 - 2x \quad dx$

14. Find the area under the graph of $y = \frac{1}{x^2}$, above the x-axis and over the interval [1, 3].

15. A company has a marginal profit function given by $P'(x) = -2x + 980$. This means that the rate of change of total profit with respect to the number of units x produced is $P'(x)$. Find the total profit from the production and sale of the 101st unit through the 800th unit of the product.

16. For the function whose graph is shown below, is $\int_a^b f(x) \, dx$ positive, negative, or zero?



Answers to Review Problems

1. (a) –2 (b) $+\infty$ (c) 0 (d) 2 (e) 0 (f) 1 (g) x = –4, 0, 2 (h) x = –4, 0, 1, 2, 3 (i) (1, 3)
   (j) $(-\infty,-4) \cup (-4,0) \cup (3,\infty)$ (k) $(-4,0) \cup (3,\infty)$ (l) $(-\infty,-4)$
2. (a) 1/3 (b) ∞ (c) 1
3. $f'(x) = \lim_{h \to 0} \frac{(3(x+h)^2 - 1) - (3x^2 - 1)}{h} = 6x$
4. (a) $f'(x) = (x+2)^2(x-3)^4(8x+1)$ (b) $f'(x) = \frac{3x}{\sqrt{3x^2+1}}$ (c) $f'(x) = \frac{e^{2x}(2x-1)}{6x^2}$

   (d) $f'(x) = (\ln 3)3^x - \frac{1}{3}x^{-2/3}$ (e) $f'(x) = \frac{1}{2(x+1)}$ (f) $f'(x) = \frac{5(2x-1)}{x^2 - x + 3}$ (g) $f'(x) = \frac{3}{x \ln 2}$

5. (a) $\frac{dA}{dr} = 2\pi r$ (b) $A'(.2) = .4\pi$ (c) When the radius of the wound is .2cm, the area of the wound is changing at the rate of $.4\pi$ square centimeters per centimeter.

6. (a) HA: $y = 1$, VA: $x = 2$, $x = -2$

   (b) $f'(x) = \frac{-8x}{(x^2-4)^2}$; x = 0, 2, –2 (c) f has a rel. max at x = 0

   (d) $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}$; concave up: $(-\infty,-2) \cup (2,\infty)$, concave down: (–2,2)

   (e) no inflection points, since the points at which the second derivative changes signs are vertical asymptotes.

7. $\frac{dy}{dx} = 2xe^{x^2}$; relative minimum when x = 0.
8. (a) 0 (b) max value $A_0 = 100$, at t = 0 (c) No. According to this model, the amount approaches, but never reaches 0.
9. Charge \$83.33 for a max profit of \$7334.80
10. (a) $N(1) = 1000$ units (b) $N'(a) = \frac{200}{a}$, $N'(10) = 20$ (c) $\lim_{a \to \infty} N'(a) = 0$; No.
11. (a) $P(t) = 60e^{.0037t}$ (b) $P(9) \approx 62.03$ million (c) $t \approx 187.34$ years

12. (a) $F(x)=\frac{1}{4}x^4-\frac{1}{2}x^2+2x+C$ (b) $F(x)=\frac{1}{3}x^3+2x^2+4x+C$

(c) $F(x)=\frac{2}{3}(x-1)^{3/2}+C$ (d) $F(x)=\frac{1}{5}e^{5x}-5x+C$

(e) $F(x)=\ln|x|-\frac{1}{a+1}x^{a+1}+C$ (f) $F(x)=52x^{1/4}+C$

13. (a) $\int_0^9 \sqrt{x}\ \ dx=\frac{2}{3}x^{3/2}\Big|_0^9=\frac{2}{3}(27-0)=18$

(b) $\int_1^4 \frac{2}{x}\ dx=2\ln|x|\Big|_1^4=2\ln 4$

(c) $\int_{-2}^3 7-2x\ \ dx=7x-x^2\Big|_{-2}^3=(21-9)-(-14-4)=30$

14. $A=\int_1^3 \frac{1}{x^2}\ dx=-\frac{1}{x}\Big|_1^3=-\frac{1}{3}-(-1)=\frac{2}{3}$

15. $P(x)=\int_{100}^{800} -2x+980\ \ dx=-x^2+980x\Big|_{100}^{800}=144{,}000-88{,}000=\$56{,}000$

16. approximately 0 (may be slightly negative)

I would also look over all your old exams, previous chapter reviews and the reviews at the end of every chapter.