5.6 VOLUME

In finding the area of a region, we drew an arbitrary representative rectangle. Keeping with the same idea, if we revolve a rectangle around a line, it forms a cylinder, as shown below.

Example: What is the volume of the cylinder shown if the height of the rectangle is considered R and the width of the rectangle is dx?

Just like we did in finding the area, as we increase the number of rectangles to infinity, the width of each rectangle becomes infinitely small and we denote this dx (if it is a vertical strip) or dy (if it is a horizontal strip). We then use an integral to sum the volume of every one of these infinitely thin cylinders. This concept leads to the following:

The Disc Method

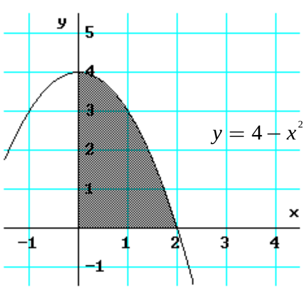
To find the volume of a solid of revolution with the *disc method*, use one of the following;

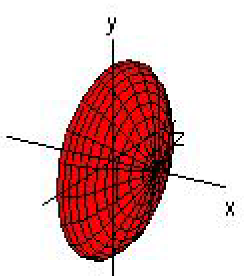
HORIZONTAL AXIS OF REVOLUTION

$$V = \pi \int_a^b \left[R(x)\right]^2 \, dx$$

where $R(x)$ is the "height" of your representative rectangular strip.

Example: Draw an appropriate rectangular strip and find the volume of the solid formed by revolving the region about the x – axis.





Example: A region is bounded by the graphs $y = e^x$, $x = -2$, and $x = 5$. Find the volume of the solid generated by rotating this region about the x-axis.

Example: [p517 #22] Consider the function $y = \frac{1}{x}$ over the interval $[1, \infty)$. We showed in section 5.3 that the area under the curve does not exist; that is $\int_1^\infty \frac{1}{x}\,dx$ diverges. Find the volume of the solid of revolution formed by rotating about the x-axis the region under the graph of $y = \frac{1}{x}$ over the interval $[1, \infty)$.

… An interesting (depending on your point of view) fact … this solid is sometimes referred to as Gabriel's horn. The paradox of Gabriel's horn lies in the fact that the surface are of the horn can be shown to be $2\pi \int_1^\infty \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\,dx$.

However, this integral diverges, which means that while we were able to find the volume in the previous example, the surface area does not exist. This is equivalent to a can of paint that holds a finite volume but, when full, does not hold enough paint to paint the outside of the can.