

5.3 IMPROPER INTEGRALS

Example: Find $\int \frac{1}{x^2} dx$. This is an example of an _____ integral.

Example: Evaluate $\int_1^5 \frac{1}{x^2} dx$. This is an example of a _____ integral.

Example: Evaluate $\int_1^b \frac{1}{x^2} dx$.

In this section the idea of an improper integral is introduced. One type of improper integrals has one (or both) of the limits of integration as infinity (or negative infinity). An example of an improper integral would be

$$\int_1^{\infty} \frac{1}{x^2} dx.$$

A Short-Study on Infinite Limits

Use your calculator, intuition, or divine intervention to answer the following questions.

Example: As x approaches infinity, what does $\frac{1}{x}$ approach?

Example: As x approaches infinity, what does \sqrt{x} approach?

Example: As x approaches infinity, what does e^x approach?

Example: As x approaches infinity, what does $\ln x$ approach?

Example: As x approaches infinity, what does e^{-x} approach?

For each of these we say $\lim_{x \rightarrow \infty} f(x) = \dots$. If the function approaches a real number, we say the limit exists.

Example: Try integrating $\int_1^{\infty} \frac{1}{x^2} dx$ the same way you would integrate any other definite integral.

What (if anything) seems to be the problem?

Example: Consider again this integral ... $\int_1^b \frac{1}{x^2} dx$ (see answer from other side) ... Find $\lim_{b \rightarrow \infty} \left(\int_1^b \frac{1}{x^2} dx \right)$.

This last example illustrates how we will determine improper integrals.

1. Replace the limit of ∞ with any variable other than the variable in the integral.
2. Evaluate the definite integral with the variable you chose in step 1.
3. Evaluate the limit of your answer from step 2 as the variable you chose approaches infinity.

Example: Evaluate $\int_2^{\infty} \frac{1}{x} dx$.

Example: Evaluate $\int_0^{\infty} 4e^{-4x} dx$

Improper integrals which have a limit which exists (and therefore an answer) are said to be _____.

Improper integrals which have a limit which does not exist are said to be _____.