4.6 Techniques of Integration: Integration by Parts

When you integrate, you find an antiderivative, or you "undo" the derivative. When you used the substitution method, you were undoing a derivative that involved the Chain Rule. In this section we will learn how to undo a derivative that involved the Product Rule.

First, recall the way we differentiate functions using the Product Rule and differential notation.

$$(uv)' = u\,dv + v\,du$$

Using the Product Rule above, we will develop a "formula" for Integration by parts.

Begin by taking the Integral of both sides.

Then rearrange the equation, solving for $\int u\,dv$.

♫: You could have just as easily solved for $\int v\,du$, but it seems like every Calculus book uses the first way!

Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du$$

Here's what you do … In order to solve integrals of this type, you need to decide what to let u and dv equal. Then follow the formula in the box above. **In order to find v from dv you must integrate, so choose wisely**.

Guidelines to Choose u and dv

- Let dv be the part that you are able to integrate.
- It helps if du is simpler than u (or at least no more complicated).
- It helps if v is simpler than dv (or at least no more complicated).

L I P E T

Further guidelines to choosing u:

First choice is a natural logarithm … *L* ….
Second choice is an inverse trigonometric function … *I* …
Third choice is a polynomial … *P* …
Fourth choice is an exponential … *E* …
Lastly, chose a trigonometric function … *T* …

Since we don't deal with trigonometric functions in this course … you're guidelines are simply LPE … any creative ideas on how to remember this?

A Special Case of Integration by Parts.

Example: $\int \ln x \, dx$

Other Examples

Example: $\int xe^x \, dx$

Example: $\int \frac{\ln(x)}{x^2} \, dx$

Example: $\int x^2 \ln x \, dx$

Repeated Use of Integration by Parts

Sometimes it is necessary to use integration by parts more than once.

Example: $\int x^2 e^{3x}\, dx$

Tabular Method

The tabular method gives you an organized way to deal with repeated use of integration by parts. Not every integral that requires repeated use of integration by parts can be done using the tabular method. However, any integral that involves the following can be done using the tabular method.

1. One part of the integral can be repeatedly differentiated easily AND will eventually go to 0
2. The other part of the integral can be repeatedly integrated easily.

The last example fits these criteria.

Example: Redo the last example using the tabular method.

Example: $\int x\sqrt{x+5}\,dx$

a) Find the indefinite integral using integration by parts.

b) Find the indefinite integral using substitution.

Definite Integration with Integration by Parts

Just find the indefinite integral and then evaluate the limits of the definite integral.

Example: $\int_1^5 xe^x\,dx$ (use the indefinite integral we found earlier)