

4.5 INTEGRATION TECHNIQUES: SUBSTITUTION

Up to this point, you have only been integrating functions by recognizing the function as an antiderivative of an elementary function. Integration by substitution allows us to integrate a function that was obtained by using the chain rule to take the derivative of a composite function.

The Guess-and-Check Method

Recall the way we differentiate composite functions using the Chain Rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Using the chain rule to find a derivative gives us a product of two functions. If we can recognize that one of these functions is the derivative of the inside of the other function, then we can undo the chain rule and find the antiderivative.

Example: Just to refresh ... use the Chain Rule to find the derivative of each function. Do not simply.

a) $f(x) = (3x^4 - 5)^{12}$

b) $g(x) = e^{x^3}$

Example: Using the patterns above, find the antiderivative of the following functions.

a) $\int (x^2 - 1)^3 2x \, dx$

b) $\int 3x^2 \sqrt{x^3 + 2} \, dx$

c) $\int x(x^3 - 5) \, dx$

What problem did you find with the part (c)? How did you correct this problem?

When we try to integrate more complicated functions, it helps to formalize this guess – and – check process.

To Make a Substitution

Let u be the “inside function” and using differential notation,
 $du = u'(x) dx$

It is important to note, that with substitution, the goal is to substitute ALL values of the integrand with either u or du . Any “extras” must be accounted for, or substitution will NOT work!

Indefinite Integration with Substitution

Example: $\int x^3 \sqrt{x^4 + 2} dx$

Example: $\int \frac{x}{x^2 + 2} dx$

Example: $\int \frac{2x}{\sqrt{x^2 + 6}} dx$

Example: $\int \frac{e^x}{e^x + 4} dx$

Example: $\int \frac{\ln x}{x} dx$

Example: A firm has the marginal-demand function $D'(x) = \frac{-2000x}{\sqrt{25-x^2}}$. Find the Demand function given that $D = 13,000$ when $x = \$3$ per unit.

Definite Integrals with Substitution

There are two ways to evaluate a definite integral using substitution (without using your calculator).

#1: Find the indefinite integral (like we just did) and use the original limits of integration.

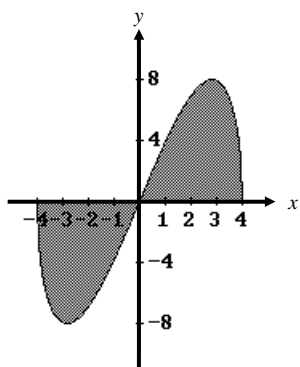
#2: Once you determine what you are going to let u be ... use that to change the limits of integration.
(This means you don't have to change everything back to x 's before you evaluate the integral.)

Example: Evaluate $\int_0^1 x\sqrt{1-x^2} dx$.

a) Use Method #1:

b) Use Method #2:

Example: Find the AREA of the region enclosed by the curve $y = x\sqrt{16-x^2}$ and the x -axis as shown below.



Example: What is the average value of $y = x^2\sqrt{x^3+1}$ on $[0, 2]$?

A) $\frac{26}{9}$

B) $\frac{52}{9}$

C) $\frac{26}{3}$

D) $\frac{52}{3}$

E) 24

Example: Oil is leaking out of a ruptured tanker at the rate of $r(t) = 50e^{-0.02t}$ thousand liters per minute.

a) At what rate, in liters per minute, is oil leaking out at $t = 0$? At $t = 60$?

b) How many liters leak out during the first hour?