4.3 AREA AND DEFINITE INTEGRALS

Example: Suppose the velocity of a particle in cm/sec is given by $v(t)=6t$, where t is in seconds. Graph $v(t)$ below.

Example: Find the area under the curve from $t = 0$ to $t = 3$. What does this area represent?

Example: Find $\int 6t\, dt$.

Example: Earlier in the year, we determined that velocity was the derivative of position. So, the antiderivative of the velocity function would be the position function, $s(t)$. What does $s(3) - s(0)$ represent?

Example: Using your answer to $\int 6t\, dt$ for the function of $s(t)$, find $s(3) - s(0)$. What do you notice?

The Fundamental Theorem of Calculus [The Evaluation Part]

If f is continuous at every point of $[a, b]$,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where $F(x)$ is an antiderivative of $f(x)$.

We read this as "the integral of $f(x)$ from a to b", where a is the Lower Bound and b is the Upper Bound.

Example: $\int_{-1}^{3} \left(x^2 - 3x + 2\right) dx$

Using Definite Integrals as Area

Ok … back to the idea of Area. We can define the **area under the curve $y = f(x)$ from a to b** as an *integral* from a to b AS LONG AS THE CURVE IS ABOVE THE X-AXIS AND CAPABLE OF BEING INTEGRATED on the closed interval $[a, b]$.

Example: Consider the function $f(x) = 3 - x$. Sketch a graph of this function.

a) What is the "area" between the curve and the x – axis between $x = 4$ and $x = 8$?

b) Evaluate $\int_4^8 (3 - x)\, dx$

c) Explain the difference between thinking of a definite integral as area under a curve in the first two examples and this example.

Example: The graph of f shown below consists of line segments and a semicircle. Evaluate each definite integral.

a) $\int_0^2 f(x)\, dx$

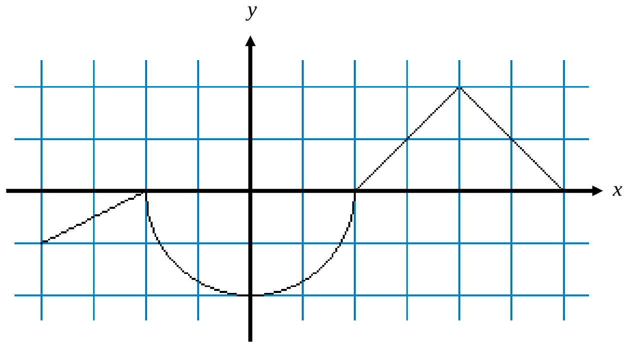
b) $\int_2^6 f(x)\, dx$

c) $\int_{-4}^2 f(x)\, dx$

d) $\int_{-4}^6 f(x)\, dx$

e) $\int_{-4}^6 |f(x)|\, dx$

f) $\int_{-4}^6 [f(x) + 2]\, dx$



Example: Kitchens-to-Please Contracting determines that the marginal cost, in dollars per foot, of installing x feet of kitchen countertop is given by $C'(x) = 8x^{-1/3}$. Find the cost of installing an extra 14 feet of countertop after 50 feet have already been ordered.

The FnInt Function of your TI – 83+

By this point, hopefully you understand the following concepts:

1. The limit of a Riemann Sum is used to define a Definite Integral
2. A Definite Integral can be used to find the Area under a curve if the curve is above the x – axis, and if the curve is below the x – axis the value of the definite integral is "negative area" … even though no one in their right mind would ever actually use that phrase in a math class if they wanted to be taken seriously!
3. Since the Definite Integral can be thought of as Area, you can draw a picture and use geometric formulas to find the areas.

BUT … what happens if you were asked right now, this instant, today to find the definite integral of a function that doesn't lend itself to nice geometric shapes?

The good news for now, is you don't even have to worry about how to do these by hand! You get to use your calculator!

<u>How to Use Your Calculator to Find a Definite Integral:</u>

The syntax for using your calculator is as follows: fnInt(*function*, *x*, *lower bound*, *upper bound*)

1. Press [MATH]
2. Press 9: fnInt(
3. Enter the function followed by [,]
4. Press [X,T,Θ,*n*] followed by [,]
5. Enter the lower bound followed by [,]
6. Enter the upper bound
7. Close the parenthesis

Example: Evaluate $\int_1^5 5x^2 \, dx$.

Example: Evaluate $3+2\int_2^3 e^{2x} - 5x^{-1/3} \, dx$

You can also do the same thing from the graphing screen.

Example: Graph $y=\sqrt{x}$ on a standard viewing window. Evaluate $\int_1^8 \sqrt{x} \, dx$.

Press [2nd][TRACE] (CALC), 7: ∫ f(x)dx, enter Lower Bound as 1, enter Upper Bound as 8.

Compare this to using to the previous method.