

4.2 AREA, ANTIDERIVATIVES, AND INTEGRALS

In the last section we found that the area under the curve of a rate of change was equal to the total amount of change. In the next few lessons we are going to connect this idea of area under the curve to the concept of *Antiderivatives*.

As the word implies, finding an antiderivative is simply undoing the derivative.

Example: Suppose that $f'(x) = 2x + 3$. What is a possible equation for the original function $f(x)$?

Example: Find some other choices for the original function $f(x)$? How do we know which one is the one we want?

Finding the antiderivative in calculus is also called finding the _____.

Of course, nothing in mathematics would be complete without having notation.

We use the notation $\int f(x) dx$ to describe the indefinite integral of $f(x)$, and the solution is given by

$$\int f(x) dx =$$

, where _____ is the antiderivative of $f(x)$ and _____ is called the constant of integration.

Rules For Finding Antiderivatives

1. Integral of a Constant

If k is a constant, then $\int k dx = kx + C$.

Example: Find $\int 5 dx$

2. Integral of x raised to a power (other than -1).

If n is any number except -1 , then $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

Example: Find $\int x^5 dx$

Example: Find $\int \sqrt{x} dx$

Example: Find $\int \frac{1}{x^3} dx$

3. Integral of x^{-1} .

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C.$$

4. Integral of e to a power of the type kx .

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C.$$

Example: Find $\int e^{5x} dx$

5. Integral of a constant k times a function.

$$\int k \cdot f(x) dx = k \cdot \int f(x) dx$$

Example: Find $\int 7x^5 dx$

Example: Find $\int \frac{8}{x} dx$

6. Integral of a sum or difference.

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

Example: Find $\int \left(3x^5 - \frac{7}{x} + 8\sqrt{x} - 13e^{2x} \right) dx$

Earlier we found multiple answers for a function whose derivative is $f'(x) = 2x + 3$. How do we know which one we want? We have to have some sort of information about the original function. This information is called an initial condition. When given an initial condition, we are able to solve for the value of C .

Example: If $f'(x) = 3x$ and $f(2) = 6$, find $f(x)$.

Example: If $f'(x) = 7e^{3x}$ and $f(0) = \frac{5}{3}$, find $f(x)$.