4.1 THE AREA UNDER A GRAPH

There are two major components to calculus. The first is a derivative, where we start with a function and then find the rate of change. The other component is an integral. The basic principle of an integral is to start with a rate of change and find the function.

Example: Suppose a oil pump is producing 800 gallons per hour for the first 5 hours of operation. For the next 4 hours, the pumps production is increased to 900 gallons per hour, and then for the next 3 hours, the production is cut to 600 gallons per hour. Make a graph modeling this situation.

Example: The term "area under a graph" is the area between the graph and the horizontal axis. Find the area under the graph from 0 to 5 hours. What does this value represent?

Example: Find the total area under the graph for the entire 12 hours. What does this value represent?

Example: Sylvie's Old World Cheeses has found that the cost, in dollars per kilogram, of the cheese it produces is

$$c(x) = -0.012x + 6.50,$$

where x is the number of kilograms of cheese produced and $x \leq 300$.

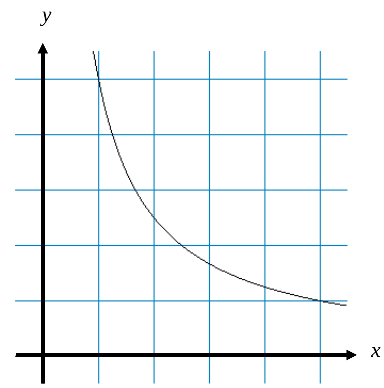
a) Draw a sketch of the cost function. Label the axes like the first example.

b) If you wanted to find the total cost of producing 400 kg of cheese how is this represented on your sketch?

c) Find the total cost of producing 400 kg of cheese.

Example: The graph of $y = \frac{5}{x}$ is shown to the right.

Why is this different than the last two examples?



Example: We can ESTIMATE the area under the curve using rectangles (or trapezoids). If you were asked to find the area under the curve from $x = 1$ to $x = 5$, by creating 4 rectangles where would you draw the rectangles?

Example: Using the left-hand endpoint, find the area under the curve $y = \frac{5}{x}$ from $x = 1$ to $x = 5$ using 4 rectangles.

Example: Repeat the last example using 5 rectangles.

Example: Soulful Scents has found that the marginal cost of producing x ounces of a new fragrance is given by

$$C'(x) = 0.0005x^2 - 0.1x + 30 ,$$

where $x \le 125$ and $C'(x)$ is in dollars. Use 5 subintervals over [0, 100] and the left endpoint of each subinterval to approximate the total cost of producing 100 oz of the fragrance.

Summation Notation

The capital Greek letter sigma, Σ, is used to indicate a sum in mathematics.

Example: Find $\sum_{i=1}^{4} 5i$

Example: Find $\sum_{i=1}^{5} \frac{5}{i}$

The area under the curve is approximated by the sum of a "bunch" of rectangles. For vocabulary purposes, these sums are called ***Riemann Sums***.

Summary of the Process of Finding a Riemann Sum: A sketch is usually helpful!

Step 1: Divide (or Partition) the interval into *n* subintervals.

Step 2: Create *n* rectangles whose base equals the width of each subinterval and whose height is determined by the function value at the left endpoint, the right endpoint, or the midpoint of the subinterval.

Step 3: Find the area of all *n* rectangles and add them together.

The conceptual KEY to this section …

The AREA UNDER THE GRAPH OF A RATE OF CHANGE = TOTAL CHANGE

Example: Finding the area under the curve of a graph that shows miles/hour gives you ____________________________.

Example: Finding the area under the curve of a graph that shows dollars/day gives you ____________________________.

Example: Finding the area under the curve of a graph that shows words/minute gives you __________________________.