

**3.5 THE DERIVATIVES OF  $a^x$  AND  $\log_a x$** 

By utilizing the rules of logarithms and implicit differentiation, you can turn an exponential equation into an equation involving logarithms that is usually easier to deal with.

*Example:* Find  $\frac{dy}{dx}$  if  $y = 2^x$ .

*Example:* Find  $\frac{dy}{dx}$  if  $y = 3^x$ .

*Example:* Make a conjecture on  $\frac{d}{dx}[a^x]$ , where  $a$  is a constant greater than 0 and not equal to 1.

We can also apply the rules of logarithms to equations involving logarithms of bases other than  $e$ .

*Example:* Using the change of base formula, find the derivative of  $y = \log_2 x$

*Example:* Using the change of base formula, find the derivative of  $y = \log_3 x$ .

*Example:* Make a conjecture:  $\frac{d}{dx}[\log_a x] =$

Derivative of  $a^u$ 

$$\frac{d}{dx}[a^x] = \ln a \cdot a^x \quad , \text{ or using the chain rule, } \frac{d}{dx}[a^u] = \ln a \cdot a^u \frac{du}{dx}$$

Derivative of  $\log_a x$ 

$$\frac{d}{dx}[\log_a x] = \frac{1}{\ln a} \cdot \frac{1}{x} \quad , \text{ or using the chain rule, } \frac{d}{dx}[\log_a u] = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot \frac{du}{dx}$$

*Example:* Find the derivatives of each of the following functions.

a)  $y = 6^x$

b)  $g(x) = x^5 (3.7)^x$

c)  $f(x) = 12^{7x-4}$

d)  $y = \log_{23} x$

e)  $g(x) = (\log_9 x)^7$

f)  $k(x) = \log(7x+3) \cdot 4^{2x^4+8}$