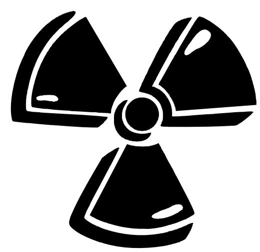
3.4 APPLICATIONS: DECAY

Example: Draw a sketch of the function $y = 30e^{5t}$.

Example: How does the graph change if we were to graph $y = 30e^{-5k}$?

The first function was an example of exponential growth and the second of exponential DECAY. In life, radioactive elements decay exponentially.

Example: **Radioactive Decay**: The rate at which a radioactive element decays (as measured by the number of nuclei that change per unit of time) is approximately proportional to the amount of nuclei present. Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident.



a) Write the differential equation that models this situation.

b) Write the function that satisfies the differential equation above.

c) Using your equation in part *b*, how long will it take for the 10 grams to decay to 1 gram?
[Pu-239 has a half life of 24,360 years]

Example: **Newton's Law of Cooling**: Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature in the surrounding medium. A detective finds a murder victim at 9 am. The temperature of the body is measured at 90.3 °F. One hour later, the temperature of the body is 89.0 °F. The temperature of the room has been maintained at a constant 68 °F.

a) Assuming the temperature, T, of the body obeys Newton's Law of Cooling, write a differential equation for T.

b) The equation that satisfies the differential equation above is

$$T = a \cdot e^{-kt} + 68 .$$

Find the value of a.

c) Find the value of k.

d) Assuming the victim had a normal body temperature of 98.6° F at the time of the murder, find when the crime occurred.