3.3 APPLICATIONS: UNINHIBITED AND LIMITED GROWTH MODELS

The following example comes directly from page 336 in your textbook, but it illustrates where the concept of uninhibited growth very well.

Example: Consider the function $y = 2e^{3x}$.

a) Find y'.

b) Rewrite y' as a function of *y*.

We say that the rate of change of *y* is proportional to the value of *y*.

The answer to part *b* above is an example of a **differential equation**.

If $\frac{dy}{dx} = k \cdot y$, then $y = y_0 e^{kx}$, where y_0 is the initial amount and *k* is the proportional constant (or growth rate).

When you solve the differential equation above for *y*, you get an exponential equation.
Example: How does the graph of $y = y_0 e^{x}$ differ from the graph of $y = e^{x}$? Draw a sketch of both graphs.

Example: Find the general form of the function that satisfies $\frac{dQ}{dt} = -.6 \cdot Q(t)$.

Example: Suppose that P_0 is invested in a savings account in which interest is compounded continuously at 3.5% per year.

a) Write the differential equation that models this situation.

b) Find the function that satisfies the equation in part *a*. Write it in terms of P_0 and 0.035.

c) Suppose that $1500 is invested. What is the balance after 1 year? After 2 years?

d) When will an investment of $1500 double itself?

Example: Finding a market niche for a specific online product, you have started you own company. Your research indicates that the number of customers using your product will increase at the rate of 10% per year.

a) Write the differential equation that models this situation.

b) Find the function that satisfies the equation in part *a*. Assume that the number of customers at $t = 0$ is 350.

c) How many customers will you have in 20 years?

d) In what period of time will the initial number of 350 customers double?

Example: A bank advertises that it compounds interest continuously and that it will double your money in 14 years. What is its annual interest rate?

The "Rule of 70"

The Rule of 70 says that the amount of time it takes to double your money is approximately 70 divided by the interest rate. Let's see why …

Example: In general if $\frac{dy}{dx} = k \cdot y$, then ________________. If you double your money, then y = _______.

Solve the equation above using a value of y that is double what you started with.

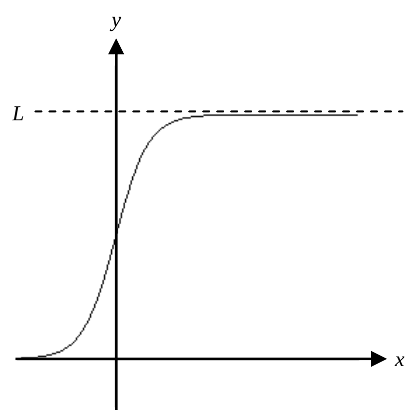
Go back to the examples on the previous page … do the answers match with the "rule of 70"?

Logistic Growth Functions

Most things in life are unable to sustain an uninhibited growth model. An exponential model may fit the data well for a time, but eventually many things in life (for various reasons) are limited in their growth.

A **logistic growth function** models this limited growth with the equation

$$P(t) = \frac{L}{1 + be^{-kt}}$$



Example: A ship carrying 1000 passengers has the misfortune to be wrecked on a small island from which the passengers are never rescued. The natural resources of the island restrict the growth of the population to a *limiting value* of 5780, to which the population gets closer and closer but which it never reaches. The population of the island after time *t*, in years, is approximated by the logistic equation

$$P(t)=\frac{5780}{1+4.78e^{-0.4t}}\ .$$

a) Find the population after 0 years.

b) Find the population after 1 year, 2 years, 10 years, and 25 years.

c) Find the rate of change, $P'(t)$.

d) Sketch a graph of the function.