3.2 LOGARITHMIC FUNCTIONS

What is an logarithmic function?

A logarithmic function is the inverse of an exponential function. A logarithmic function $f(x)$ is given by _____, where b is the base.

If $\log_a b = c$, then this means that _____, and vice-versa.

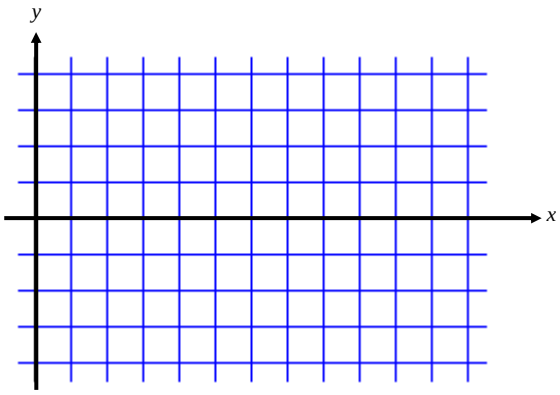
Example: Rewrite the logarithmic equation $\log_{49} 7 = \frac{1}{2}$ as an exponential equation.

Example: Rewrite the exponential equation $4^3 = 64$ as a logarithmic equation.

Example: Rewrite the equation below as an exponential equation, then use the table to graph.

$$g(x) = \log_3 x.$$

x	y
	–2
	–1
	0
	1
	2



Properties of Logarithms

Logarithm of a product:

$$\log_a (xy) = \log_a x + \log_a y$$

Inverse Properties of a logarithm:

$$a^{\log_a x} = x \qquad \log_a a^x = x$$

Logarithm of a quotient:

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Logarithm of a power:

$$\log_a \left(x^y\right) = y \cdot \log_a x$$

Other properties:

$$\log_a a = 1 \qquad \log_a 1 = 0$$

Change of Base Formula:

$$\log_a x = \frac{\ln x}{\ln a}$$

Example: If $\log_b 3 = 1.099$ and $\log_b 5 = 1.609$, then what is $\log_b 75$?

The Change of base formula from the previous page is needed when using your calculator. There are only TWO types of logarithms your calculator is capable of finding. Those with a base of 10, and those with a base of *e*.

A log with base 10 is called a _____ logarithm, and is written as _____.

A log with base *e* is called a _____ logarithms, and is written as _____.

Solving equations involving powers of e.

Example: Solve for *x* in the following equations.

a) $e^{4x} = 500$

b) $5e^{2x} - 7 = 28$

Derivative of Natural Logarithmic Functions

The derivative of $y = \ln x$ is given by $y' =$ _____.

Using the chain rule, if *u* is a differentiable function of *x* then $\frac{d}{dx}[\ln u] =$ _____.

Example: Find the derivative of each function below.

a) $f(x) = 6\ln x$

b) $g(x) = \ln(6x)$

c) $k(x) = \ln\left(x^2 + 3x + 7\right)$

d) $r(x) = e^{3x} \cdot \ln(2x + 5)$