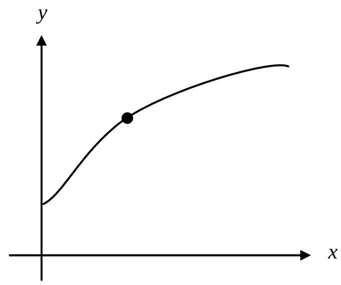
2.6 MARGINALS AND DIFFERENTIALS

Example: Draw a tangent line at the indicated point on the function below.



Suppose the original function was a profit function. Can we use the tangent line you drew to estimate the profit?

Label the initial point (x, $P(x)$). If we increased x by 1, what is the actual change in the profit?

Now consider the tangent line …

The slope of the tangent line is given by _____________, and slope of a line is given by _________.

If we consider a change in x of 1, then the slope of the tangent line = the change in y ($P'(x) = \Delta y$).

This is the underlying principle in the following definitions. If x changes by 1 unit, then the change in y is approximately the value of the derivative at the original x.

Marginal Cost

The marginal cost at x, given by $C'(x)$, is the approximate cost of the (x + 1)st item.

Meaning, $C(x+1) \approx C(x) + C'(x)$

Marginal Revenue

The marginal cost at x, given by $R'(x)$, is the approximate cost of the (x + 1)st item.

Meaning, $R(x+1) \approx R(x) + R'(x)$

Marginal Profit

The marginal cost at x, given by $P'(x)$, is the approximate cost of the (x + 1)st item.

Meaning, $P(x+1) \approx P(x) + P'(x)$

Example: Suppose that the daily cost, in dollars, of producing x radios is $C(x) = 0.002x^3 + 0.1x^2 + 42x + 300$, and currently there are 40 radios produced daily.

a) What is the current daily cost?

b) What would the additional daily cost of increasing production to 41 radios daily?

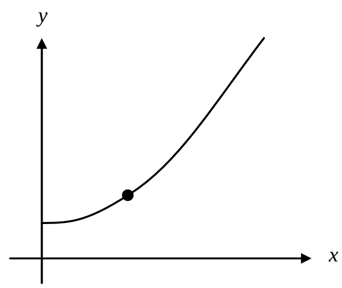
c) What is the marginal cost when x = 40?

d) Use the marginal cost to estimate the daily cost of increasing production to 42 radios daily.

Differentials

Approximations aren't exact! (Aren't you glad you woke up this morning to hear that enlightening bit of information?!) If we use a tangent line to approximate a curve, it gives us a good estimate, <u>as long as we don't go too far away from the original point</u>. In the previous examples we moved only 1 space away.

The distance we move away the original point can be thought of as Δx. Let's look at the function below.



Label the given point (x, $f(x)$). Draw the tangent line to the curve. The slope of this tangent line is given by __________.

Move to the right Δx from the original point. If the ACTUAL change in the y-values is given by Δy, how does the change in y compare to the slope of the tangent line?

When Δx is very small (infinitesimally small) we say $\Delta x = dx$ (the differential of x). Is $\Delta y = dy$?

Since dy is the approximate change in the y values when x is changed a small amount, we can use differentials to estimate the change in other problems if we know the small change in x.

Example: Find the differential dy when $dx = 0.01$ and $x = 2$, if $y = x^5 - 4x^3$. Explain what you've found.

Example: Use differentials to approximate $\sqrt{4.2}$.