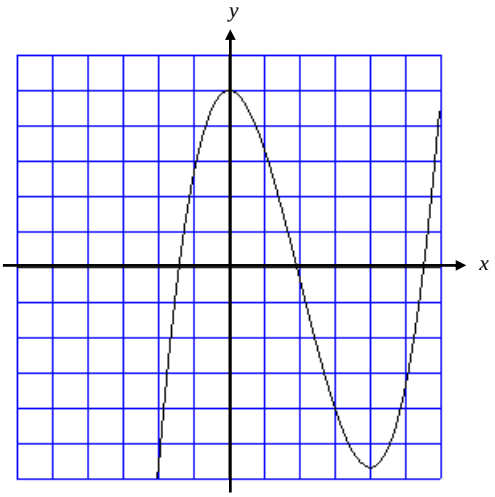
2.2 USING SECOND DERIVATIVES TO FIND MAXIMUM AND MINIMUM VALUES AND SKETCH GRAPHS

Example: The function $f(x)=\frac{1}{3}x^3-2x^2+5$ is shown below.



a) Find $f'(x)$ and graph on the second grid.

When f is increasing, the graph of f' is _____.

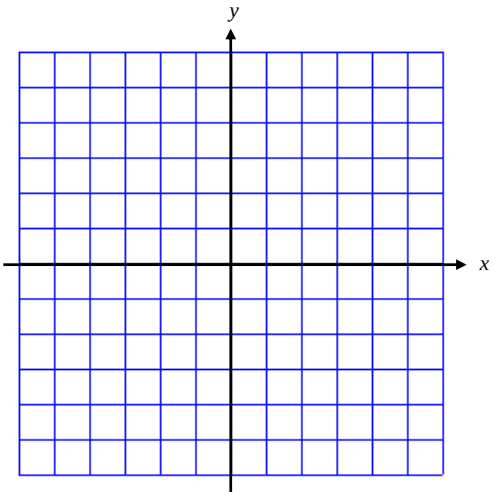
When f is decreasing, the graph of f' is _____.

There is a relative maximum on f when f'

_____.

There is a relative minimum on f when f'

_____.



b) Find $f''(x)$ and graph on the third grid.

Concavity deals with the curvature of a graph.

A concave upward graph looks like 

A concave downward graph looks like 

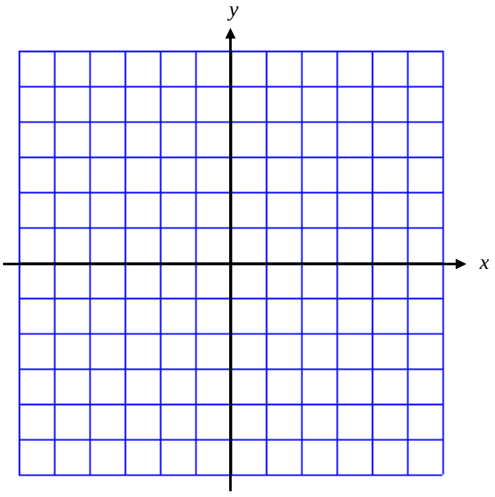
When f is concave up, f'' is _____ and

f' is _____.

When f is concave down, f'' is _____ and

f' is _____.

.



c) Look at the critical numbers of f.

If c is a critical number of f then, $f(c)$ is a **maximum** if $f''(c)$ ____ 0.

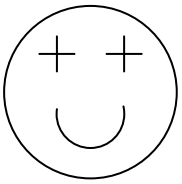
If c is a critical number of f then, $f(c)$ is a **minimum** if $f''(c)$ ____ 0.

This is known as the ***SECOND DERIVATIVE TEST***.

The concavity test can be remembered with the following pictures … keep in mind these are NOT to be used for justification.

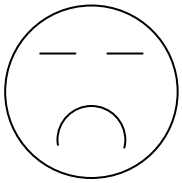
f'' positive

f Concave UP



f'' negative

f Concave DOWN



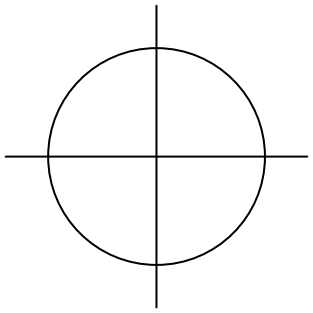
Example: Label each quadrant below with one of the following descriptions:

i) Increasing and Concave Up

ii) Increasing and Concave Down

iii) Decreasing and Concave Up

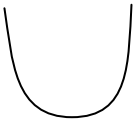
iv) Decreasing and Concave Down.



Points of Inflection

A point of inflection is a point on the graph where the concavity changes.

Example: Estimate the point(s) of inflection on each curve, if they exist.



Points of inflection occur where concavity changes, and concavity changes when _____.

We look for points of inflection in two places.

1.

2.

REMEMBER … in order for a point of inflection to exist, the CONCAVITY MUST CHANGE

Example: Find the points of inflection of $g(x) = -2x^3 + 6x^2 - 3$.

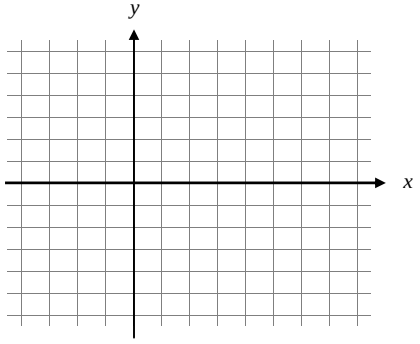
Example: Find the points of inflection of $h(x) = x\sqrt{4-x^2}$.

Example: Let f be a function that is continuous on the interval [0, 4]. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table below, where DNE indicates that the derivatives of f do not exist at $x = 2$.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

b) On the axes provided, sketch the graph of a function that has all the characteristics of f.



Example: Sketch $f(x) = x^3 - 6x^2 + 9x + 1$

a) Find the critical points using the first derivative.

b) Find the *possible* points of inflection using the second derivative.

c) Using both the critical points and points of inflection, make a sign chart with the first and second derivatives.

d) Use your sign chart to determine

i) Which values of x (if any) give you a relative minimum, relative maximum or point of inflection.

ii) Whether your function is increasing, decreasing, concave up, or concave down.

e) Sketch your function using ALL this information.