

**1.5 DIFFERENTIATION TECHNIQUES: THE POWER AND SUM-DIFFERENCE RULES**

Before we begin ... a little refresher ...

*Example:* Rewrite each term as a power of  $x$ .

a)  $\sqrt{x}$

b)  $\sqrt[5]{x^2}$

c)  $\frac{1}{x^2}$

d)  $\sqrt[3]{x} - \frac{1}{x^2} + x^3$

In the last section we discovered the tangent line of a curve can be found by finding the limit as  $h$  approaches 0 of a difference quotient. Using this method in more general terms allows us to prove the following rules, which make taking a derivative much easier. Please don't throw things at the instructor. ☺

First ... a few notes about notation ...

*Notation used for the derivative of a function ...*

$f'(x)$  ... read “ $f$  prime of  $x$ ”

$y'$  ... read “ $y$  prime”

$\frac{dy}{dx}$  ... read “the derivative of  $y$  with respect to  $x$ ”

$\frac{d}{dx}[f(x)]$  ... read “the derivative of the function  $f(x)$  with respect to  $x$ ”

$\left. \frac{dy}{dx} \right|_{x=7}$  ... read “the derivative of  $y$  with respect to  $x$  when  $x = 7$ ”

**Rule #1 Derivative of a Constant Function**

If  $c$  is any constant value, then  $\frac{d}{dx}[c] = 0$

This should not be too earth shattering to you, since the slope of a constant function is always 0!

*Example:* Let  $f(x) = 5$ . Find  $f'(x)$ .

**Rule #2 Power Rule**

If  $n$  is any number, then  $\frac{d}{dx}[x^n] = n \cdot x^{n-1}$ , provided  $x^{n-1}$  exists.

*Example:* Let  $f(x) = x^5$ . Find  $f'(x)$ .

*Example:* Let  $y = \sqrt[3]{x^2}$ . Find  $\frac{dy}{dx}$ .

*Example:* Find  $\frac{d}{dx}\left[\frac{1}{x^4}\right]$ .

Rule 3: *The Constant Multiple Rule*

If  $c$  is a constant, then

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)].$$

*Example:* Let  $y = 5x^7$ . Find  $\frac{dy}{dx}$ .

*Example:* Let  $g(x) = \frac{4}{5x^3}$ . Find  $g'(x)$ .

Rule 4: *The Sum and Difference Rule*

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \quad \dots \quad \text{AND} \quad \dots \quad \frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$$

*Example:* Let  $y = x^3 + 4x^2 - 2x + 7$ . Find  $y'$ .

*Example:* Let  $g(x) = \frac{3}{(-2x)^4} - \frac{x}{2} + \frac{1}{4}$ . Find  $\frac{d}{dx}[g(x)]$ .

*Example:* Find the equation of the tangent line to the function  $f(x) = 4x^3 - 6x + 5$  when  $x = 2$ .

*Example:* Find all points where the graph of  $y = x^4 - 5x^3 - 3x^2 + 13x + 10$  has a horizontal tangent line.

*Example:* Let  $h(x) = (x^2 + 1)(2x - 5)$ . Find  $h'(x)$ .

*Example:* The volume of a cube with sides of length  $s$  is given by  $V = s^3$ . Find  $\frac{dV}{ds}$  when  $s = 4$  centimeters.