

1.3 AVERAGE RATES OF CHANGE

Average Rates of Change

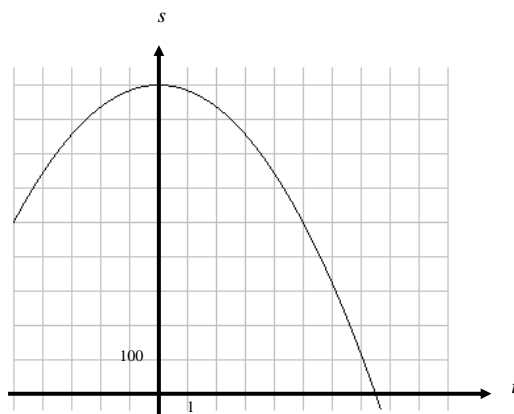
Example: Remember this example? ... Wile E. Coyote, once again trying to catch the Road Runner, waits for the nastily speedy bird atop a 900 foot cliff. With his Acme Rocket Pac strapped to his back, Wile E. is poised to leap from the cliff, fire up his rocket pack, and finally partake of a juicy road runner roast. Seconds later, the Road Runner zips by and Wile E. leaps from the cliff. Alas, as always, the rocket malfunctions and fails to fire, sending poor Wile E. plummeting to the road below disappearing into a cloud of dust.



Let's look at this problem from a graphical perspective. The equation that models Wile E.'s height at any time t is given by

$$s(t) = -16t^2 + 900$$

A graph of this equation is shown below.



Example: What is $s(0)$? Explain its meaning in the context of this problem.

Example: What is $s(5)$? Explain its meaning in the context of this problem.

Graphically, the average rate of change is simply the _____ between two points.

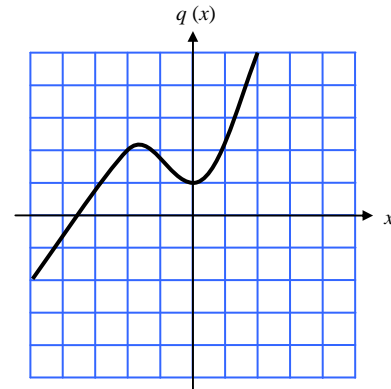
Example: Draw the line on the graph, and find the average rate of change from $t = 0$ to $t = 5$. Indicate units of measure.

A _____ is a line through any two points on a curve.

Just like we did in this example, we can always think of the **average rate of change** as the slope of a secant line.

Example: Use the graph of $q(x)$ to answer the following questions.

- What is $q(0)$?
- What is $q(-5)$?
- What is the average rate of change from $x = -5$ to $x = 0$?



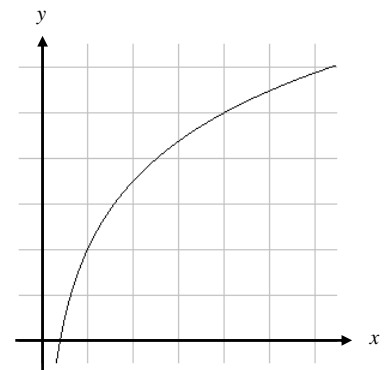
Example: Suppose that Sport Stylz Inc. determines that the cost, in dollars, of producing x cellphone-sunglasses is given by

$$C(x) = -0.05x^2 + 50x$$

Find $\frac{C(301) - C(300)}{301 - 300}$, and interpret the significance of this result to the company.

Example: Identify or sketch each of the quantities on the figure to the right.

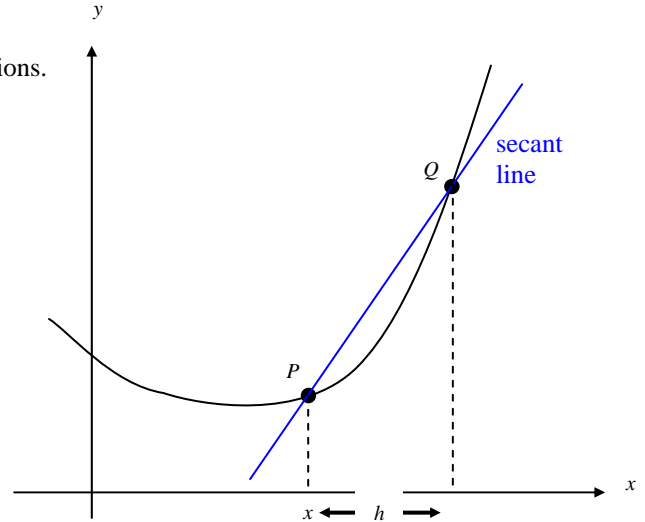
- $f(1)$ and $f(4)$
- $f(4) - f(1)$
- $y = \frac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$



Difference Quotients

Example: Use the picture to the right to answer the following questions.

- What are the coordinates of P ?
- What are the coordinates of Q ?
- What is the slope of the secant line from P to Q .



The answer to part *c* above is called a **difference quotient**. Our immediate goal is to simplify a difference quotient.

♫: In general, a difference quotient will be simplified when you have cancelled the h in the denominator.

Example: Consider the function $f(x) = 3x + 2$.

- Write the difference quotient.
- Simplify the difference quotient.

Example: Consider the function $k(x) = \frac{7}{x}$

- Write the difference quotient.
- Simplify the difference quotient.

