

1.2 ALGEBRAIC LIMITS AND CONTINUITY

For many “well behaved” functions, we can find the limits simply by direct substitution of the x -value. More on “well behaved” functions later.

Direct Substitution will yield one of three general results ...

i) A real number ... that’s your limit ... write it down ... done!

ii) A non-zero value divided by 0 ... this DOES NOT EXIST ... which means your limit doesn’t exist ... done!

iii) $0/0$... $0/0$ is an indeterminate form ... it doesn’t tell you anything ... NOT done! ... DO SOMETHING ELSE

For case *iii*, try factoring the numerator and denominator (or both) to find the like factors.

After cancelling the like factors, try direct substitution again.

Example: Here are the three limits we first found in section 1.1. Find each limit by direct substitution.

a) $\lim_{x \rightarrow 2} (2x - 3) =$

b) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} =$

c) $\lim_{x \rightarrow 2} \left(\frac{1}{x - 2} + 3 \right) =$

Check these with your previous graphs and tables.

Example: $\lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} =$

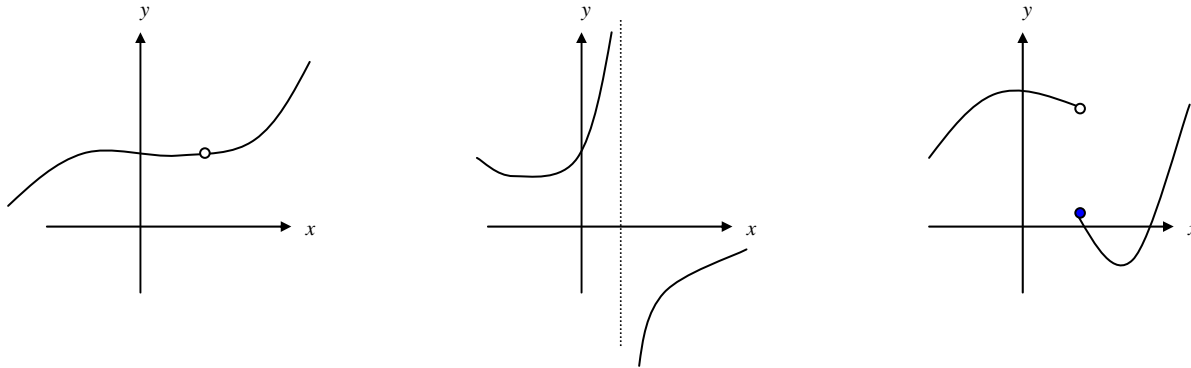
When factoring doesn’t work! ... what do I do?

Example: $\lim_{h \rightarrow 1} \frac{\sqrt{h+3} - 2}{h - 1} =$

Continuity

“Well behaved” functions allowed us to find the limit by direct substitution. “Well behaved” functions turn out to be continuous functions.

In non – technical terms, a function is continuous if you can draw the function “without ever lifting your pencil”. The following graphs demonstrate three types of discontinuous graphs.

*Definition of Continuity*

A function $f(x)$ is continuous at $x = a$ if ALL three of the statements below are true.

- 1) $f(a)$ exists
- 2) $\lim_{x \rightarrow a} f(x)$ exists
- 3) $\lim_{x \rightarrow a} f(x) = f(a)$

Example: Look again at the three pictures above. Which part(s) of the definition of continuity fail for each picture?

Example: Is the function $h(x) = \begin{cases} \frac{1}{3}x + 4 & ; x \leq 3 \\ 2x - 5 & ; x > 3 \end{cases}$ continuous at $x = 3$? Why or why not?

Example: Is the function $g(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & ; x \neq 2 \\ 5 & ; x = 2 \end{cases}$ continuous at $x = 2$? Why or why not?