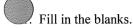
KEY

# 7.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.



a) Integrating velocity gives <u>Pisplacement</u>

b) Integrating the absolute value of velocity gives \_\_Distance Traveled ...

c) New position = Original Position + Displacement.

9

[CALCULATOR]

2. A particle moves along the x – axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time t = 0, the particle is at position x = 1.

a) Find the acceleration of the particle at time t = 2. Is the speed of the particle increasing at t = 2? Why or why not?

$$\alpha(2) = v'(2) = 1.587586682$$
  $4 v(2) = -2.72789228$ 

b) Find all times t in the open interval 0 < t < 3 when the particle changes direction. Justify your answer.

c) Find the total distance traveled by the particle from time t = 0 until time t = 3.

$$\int_{0}^{3} |v(t)| dt = 4.333818626$$

d) During the time interval  $0 \le t \le 3$ , what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

time | Position = 
$$1 + \int_{a}^{a} (t) dt$$

0 | 1

2.506283 -2.2654828

3 | -1.19714741

Show the work that leads to your answer.

$$\frac{time | Position = 1 + \int v(t) dt}{0} \int_{0}^{2.5066283} v(t) dt = -3.2654828$$

$$2.5066283 - 2.2654828$$

$$\int_{0}^{3} v(t) dt = -2.19714741$$

: The greatest distance between the particle and the origin is 2.2654828 LEFT of the origin.

3. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}$$
. people/hr

The rate at which people leave the same amusement park on the same day is modeled by function L defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}$$
. people/hr

Both E(t) and L(t) are measured in people per hour and time t is measured in hours after midnight. These functions are valid for  $9 \le t \le 23$ , the hours during which the park is open. At time t = 9, there are no people in the park.

a) How many people have entered the park by 5:00 pm (t = 17)? Round your answer to the nearest whole number.

b) The price of admission to the park is \$15\$ until 5:00 pm (t = 17). After 5:00 pm, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

that enter from 5 pm until metallott = 
$$\int_{17}^{23} E(4) dt = 1271.28276$$

$$6004(15) + 1271(11) = 404041$$

c) Let  $H(t) = \int (E(x) - L(x)) dx$  for  $9 \le t \le 23$ . The value of  $\underline{H(17)}$  to the nearest whole number is 3725.

Find the value of H'(17) and explain the meaning of H(17) and H'(17) in the context of the park.

$$H'(t) = E(t) - L(t)$$
 ... .!  $H'(17) = E(17) - L(17) = -380.28/4259$ 

H'(17) = -380.281 is the Change in the # of people in the park at 
$$t=17$$
 (5pm)

So at 5pm there are 3,725 people and the same decreasing at  $\approx$  380 people/hr.

d) At what time  $t$ , for  $9 \le t \le 23$ , does the model predict that the number of people in the park is a maximum?

: The maximum # of people in the park occurs at 15.794815 hours after milhight

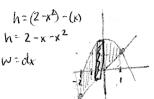
4. Complete the following questions from the textbook: page 386 #9, 12 – 16, 17, 19, 21, 31 - 36

## AP Calculus

### 7.2 Worksheet

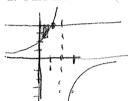
All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

Find the area of the region bounded by the graphs of  $f(x) = 2 - x^2$  and g(x) = x.



$$\frac{y-x^2=x}{0=x^2+x-2} \qquad A = \left[ \int_{-2}^{1} \left( 2-x-x^2 \right) dx \right] = \left[ 2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^{1} \\
0 = (x+2)(x-1) \\
x = -2 + x = 1$$

2. Find the area of the region bounded by the graphs of  $g(x) = \frac{4}{2-x}$ , y = 4, and x = 0.



$$h = 4 - \frac{4}{2 - x}$$

$$w = dx$$

$$\int_{0}^{1} \left( 4 - \frac{4}{2 - \kappa} \right) dx = 4x + 4 \ln |2 - \kappa| \Big|_{0}^{1}$$

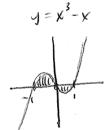
$$= \left( 4 + 4 \ln |1| \right) - \left( 0 + 4 \ln |2| \right)$$

$$= \left( 4 - 4 \ln |2| \approx |1/2| 2 |1/2| 2 |1/2| 18 \right)$$

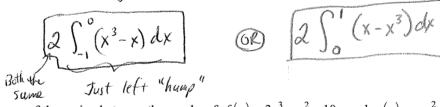
3. The area of the region bounded by the graphs of  $y = x^3$  and y = x cannot be found by the single integral

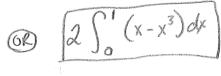
$$\int_{-1}^{1} \left( x^3 - x \right) dx.$$

Explain why this is so. Use symmetry to write a single integral that does represent the area. (Use your calculator to generate a picture)

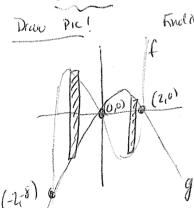


if you integrate 
$$\int_{-1}^{1} (x^2 - x) dx$$
 you get  $0$  ... but the AREA is obviously Not =  $0$ 

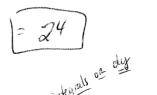




[Calculator] Find the area of the region between the graphs of  $f(x) = 3x^3 - x^2 - 10x$  and  $g(x) = -x^2 + 2x$ .



$$\int_{-2}^{6} \left[ (3x^{3} - x^{2} - 10x) - (-x^{2} + 2x) \right] dx + \int_{0}^{2} \left[ (-x^{2} + 2x) - (3x^{3} - x^{2} - 10x) \right] dx$$



There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by  $f(t) = 7te^{\cos t}$  cubic feet per hour, where t is

measured in hours since midnight. Janet starts removing snow at 6 A.M. (t = 6). The rate g(t), in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled

Show Removal
$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9 \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at  $(8 \text{ A.M.}) \neq = \%$ 

= 59.583 ft3/hr of snav is being Removed

(c) Let h(t) represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain  $0 \le t \le 9$ .

$$h(t) = \begin{cases} 0 & 0 \le t \le 6 \\ 125(t-4) & 6 \le t \le 7 \end{cases}$$
 $(125+108(t-1)) 7 \le t \le 9$ 

h(t) = Amount of SWEW Removed A When 05+ LG Jant removed 0 When 66+ L7 Jant remues snow at a Rek of 125 ft3/hr so the total amount of saw removed by for some time between 6 \$7 would be ...

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

removed at a late of 108 ft3/hr

t (hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t = 0) and 8 P.M. (t = 8). The number of entries in the box t hours after noon is modeled by a differentiable function E for 0 < t < 8. Values of E(t), in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate in hundreds of entries per hour, at which entries were being deposited at time  $t \neq 6$ . Show the computations that lead to your answer.

Approx 
$$E'(6) = \frac{E(7) - E(5)}{7 - 5} = \frac{21 - 13}{2} = 4$$
 hundreds of entries/hr

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of  $\frac{1}{8}\int_{0}^{8} E(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{8}\int_{0}^{8} E(t) dt$  in terms of the number of entries.

$$\frac{1}{8} \left[ 2\left(\frac{0+4}{2}\right) + 3\left(\frac{4+13}{2}\right) + 2\left(\frac{13+21}{2}\right) + i\left(\frac{28+23}{2}\right) \right] = \frac{1}{8} \left(85.5\right) = 10.6875$$
Integral the aug to genties (in hundreds)
Integral the collected over the 8 hrs

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled (not to Bon) by the function P, where  $P(t) = t^3 - 30t^2 + 298t - 976$  hundreds of entries per hour for  $8 \le t \le 12$ .

According to the model, how many entries had not yet been processed by midnight (t = 12)?

$$\int_{8}^{12} P(t) dt = 16$$

$$1 = 16$$

$$1 = 16$$

$$1 = 16$$

$$23 - 16 = 7 \text{ (hunched)}$$

$$1 = 10$$

$$1 = 10$$

$$1 = 10$$

$$1 = 10$$

$$1 = 10$$

$$1 = 10$$

$$1 = 10$$

(d) According to the model from part (c), at what time were the entries being processed most

quickly? Justify your answer. when is the Rock P(t) highest? MAX P(t) check endpoints of critical points. P'(t)=0 at t=9.1835034 (A) P'(t)= 3+2-60++2984

#### AP Calculus

## 7.3 Worksheet (Day 1)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The base of a solid is the region enclosed by the graph of  $y = e^{-x}$ , the coordinate axes, and the line x = 3. If all plane cross sections perpendicular to the x – axis are squares, then its volume is





B) 
$$\frac{1}{2}e^{-6}$$

C) 
$$e^{-6}$$

D) 
$$e^{-3}$$

E) 
$$1 - e^{-3}$$

$$V = \int_{0}^{3} e^{-2x} dx = \begin{bmatrix} e^{-2x} & 1 & 3 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-6} & e^{0} & e^{0} \\ -2 & -2 & -2 \\ \hline -2 & -2 & -2 \end{bmatrix}$$

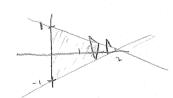
$$= \begin{bmatrix} e^{-6} & -1 \\ -2 & 1 \end{bmatrix}$$

2. [Calculator] The base of a solid S is the region enclosed by the graph of  $y = \sqrt{\ln x}$ , the line x = e, and the x-axis. If the cross sections of S perpendicular to the x – axis are semicircles, then the volume of S is

$$V = \frac{\pi}{8} \int_{1}^{6} \ln x \, dx = \frac{\pi}{8}$$

AREA OF SEMICIRCLE = 
$$\frac{1}{2}\pi R^2$$
  
=  $\frac{1}{2}\pi \left(\frac{1}{2}\int_{-\pi}^{\pi}x\right)^2$   
=  $\frac{1}{2}\pi \cdot \frac{1}{4}\cdot \ln x$   
=  $\frac{\pi}{8}\ln x$ 

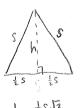
3. The base of a solid is the region bounded by the lines  $f(x) = 1 - \frac{x}{2}$ ,  $g(x) = -1 + \frac{x}{2}$ , and x = 0. If the cross sections perpendicular to the x-axis are equilateral triangles, find the volume of the solid.





= 1-
$$\frac{x}{2}$$
+1- $\frac{x}{2}$ 
= 2- $x$ 

HERROF equilateral  $\Delta = \frac{\sqrt{3}}{4}S^2$   $= \frac{\sqrt{3}}{4}(2-x)^2$ by 30-60-90 As  $= \frac{\sqrt{3}}{4}(2-x)^2$ Volume or one scale =  $\frac{\sqrt{3}}{4}(2-x)^2$  dx  $A = \frac{1}{4}S^2$   $A = \frac{\sqrt{3}}{4}S^2$ 

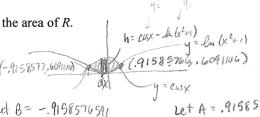


$$V = \frac{\sqrt{3}}{4} \int_{0}^{2} (2-x)^{2} dx = \frac{\sqrt{3}}{4} \int_{0}^{2} (1-4x+x^{2}) dx$$

$$= \frac{\sqrt{3}}{4} \left( 4x-2x^{2}+\frac{x^{3}}{3} \right)_{0}^{2}$$

$$= \frac{\sqrt{3}}{4} \left( 8-8+\frac{8}{3} \right) - (0-0+0)$$

$$= \frac{\sqrt{3}}{4} \cdot \frac{8}{3} = \frac{2\sqrt{3}}{3}$$



$$\int_{B}^{A} \left[ \cos x - \ln (x^{2} + 1) \right] dx = 1.167854074$$
[1.167 or 1.168 accepted]

b) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is an equilateral triangle. Write an expression involving one or more integrals that gives the volume of the solid. Do not evaluate.

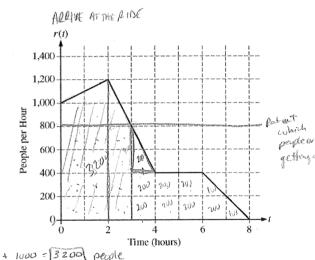


SIDE OF 
$$A = \cos x - \ln (x^2 + 1)$$
  
AREA OF  $A = \frac{\sqrt{3}}{4} \left(\cos x - \ln (x^2 + 1)\right)^2$ 

$$V = \int_{B}^{A} \frac{\sqrt{3}}{4} \left( \cos x - \ln \left( x^{2} + 1 \right) \right)^{2} dx = .3966826869$$

5. Complete the following questions from your textbook: page 406 #1, 3, 41, and 42

6. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, r(t), at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



a) How many people arrive at the ride between t = 0 and t = 3? Show the computations that lead to your answer.

$$\int_{0}^{3} (14) dt = \text{Area under curve of 1(4)} \int_{0}^{3} (1200 + 800) = 3200 + 1000 = 3200 \text{ people}$$

b) Is the number of people waiting in line to get on the ride increasing or decreasing between t = 2 and t = 3? Justify your answer.

Shoeth # of people arriving perhour from t=2 to t=3 is greater than 800 (the rate at which people are getting on the rule) the # of people in line is increasing between [i.e. r(t)>800 for t between t=2d+=3] t=2 \$ t=3.

c) At what time t is the line for the ride the longest? How many people are in line at that time?

Justify your answers. N(t) = # of people in line at time t

$$N(0) = 700$$

$$N(0) = 700 + \int_{0}^{8} (10) dx - 800(8)$$

$$\frac{r(t)=800}{\text{who } t=3}$$

N(t) = 700+ (t ((x) -800) dx

 $N(t) = 700 + \int_{0}^{10} (x) - 800) dx$   $= 700 + \int_{0}^{10} (x) dx - 800 t$   $= 700 + \int_{0}^{10} (x) dx - 800 t$  = 700 + 3200 - 2400 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 = 700 + 3200 time t at which there is no longer a line for the ride.

700 + Stronger - 800t = 0

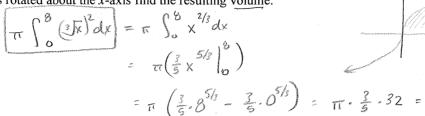
. the are 1500 neight Wille at t=3

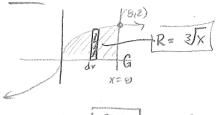
# 7.3 Worksheet (Day 2)



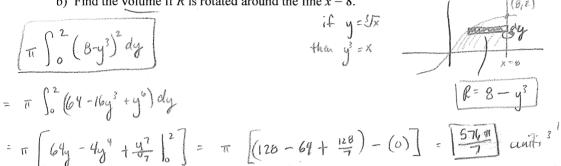
All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

- 1. Suppose region R is in the first quadrant bounded by the graphs of  $y = \sqrt[3]{x}$  and x = 8.
  - a) If R is rotated about the x-axis find the resulting volume.





b) Find the volume if R is rotated around the line x = 8.



2. The region in the first quadrant bounded by the graph of  $y = \sec x$ ,  $x = \frac{\pi}{4}$ , and the axes is rotated about the x-axis. What is the volume of the solid generated?

A) 
$$\frac{\pi^2}{4}$$
 $R = \sec x$ 
 $A = \frac{\pi^2}{4}$ 

A)  $2\pi$ 

B) 
$$\pi - 1$$
 (C)  $\pi$  units 2. D)  $2\pi$ 

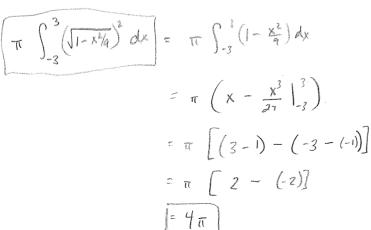
$$\pi \int_{0}^{\pi/4} \sec^{2}x \, dx = \pi \left(\tan x \Big|_{0}^{\pi/4}\right) = \pi \left(\tan^{\pi/4} - \tan 0\right)$$

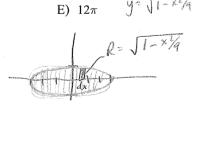
$$= \pi \left(1 - 0\right)$$

1x2 + y2=1 3. The volume of the solid obtained by revolving the region enclosed by the ellipse  $x^2 + 9y^2 = 9$  about the x-axis is

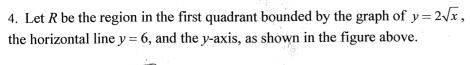
D)  $9\pi$ 

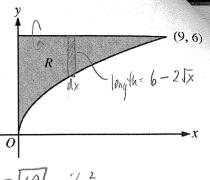
$$\left[ \frac{1}{\pi} \int_{-3}^{3} \left( \sqrt{1 - \frac{x^2}{4}} \right)^2 dx \right] = \frac{1}{\pi} \int_{-3}^{3} \left( 1 - \frac{x^2}{4} \right) dx$$





E)  $\frac{8\pi}{3}$ 





a) Find the area of 
$$R$$
.

$$A = \int_{0}^{9} (6-25x) dx = \int_{0}^{4} (6-25x) dx = 6x - 2 \cdot \frac{1}{3}x^{3/2} \Big|_{0}^{4}$$

$$= (6.9 - \frac{4}{3}.9^{3/2}) - (6-0) = 6$$

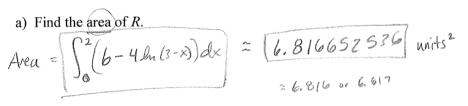
$$= 54 - \frac{4}{3}.\frac{27}{9} = 54 - 36 = [18] \text{ units}^{2}$$

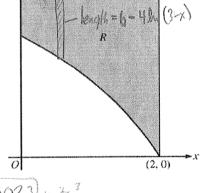
b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when 
$$R$$
 is rotated about the horizontal line  $y = 6$ .

$$V = \left( \pi \int_{0}^{q} \left( 6 - 2J_{x} \right)^{2} dx \right)$$

c) Region R is the base of a solid. For each y, where 
$$0 \le y \le 6$$
, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

5. [Calculator] In the figure above, R is the shaded region in the first quadrant bounded by the graph of  $y = 4\ln(3 - x)$ , the horizontal line y = 6, and the vertical line x = 2.





b) Find the volume of the solid generated when R is revolved about the horizontal line v = 6.

$$R = 6 - 4 \ln(3-x)$$

$$\pi \int_{0}^{2} (6 - 4 \ln(3-x))^{2} dy \approx [82.51896023] \text{ units}^{3}$$

$$\approx 82.518 \text{ or } 82.519$$

c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

- 6. [Calculator] The rate at which people enter an auditorium for a rock concert is modeled by the function R given by  $R(t) = 1380t^2 675t^3$  for  $0 \le t \le 2$  hours; R(t) is measured in people per hour. No one is in the auditorium at time t = 0, when the doors open. The doors close and the concert begins at time t = 2.
  - a) How many people are in the auditorium when the concert begins?

b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

Rate is a maximum of 
$$0 \le t \le 2$$
 at endpaints ( $t = 0 \le t \le 2$ ) or when  $R'(t) = 0$ 

$$R(t) = 2760 t = 2025 t^{2} = 0$$

$$R(t) = 2760 t = 2025 t^{2} = 0$$

$$R(t) = 2760 t = 2025 t^{2} = 0$$

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$$R(t) = 2760 t = 2025 t^{2} = 0$$

$$R(t) = 2760 t = 2025 t^{2} = 0$$

c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by w'(t) = (2-t)R(t). Find w(2) - w(1), the total wait time for those who enter the auditorium after time t = 1.

Key... understand that 
$$\int_{-1}^{2} w'(t) = w(2) - w(1)$$
 by Fundamental theorem of calculus
$$= \int_{-1}^{2} (2-t)R(t)dt \approx 387.5 \text{ hours}$$

d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part c.

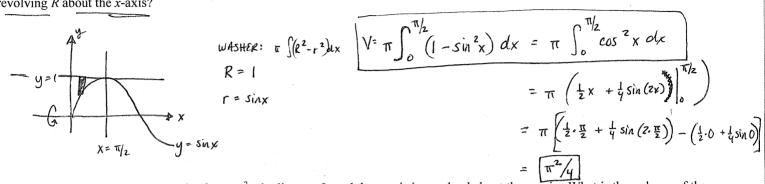
total wait time
$$\int_{0}^{2} (z-t) R(t) dt = \frac{760}{980} \approx .7755102041 \text{ hours of average wait Per person.}$$
answer from part (a).
$$\frac{2}{775} \text{ or .776 hrs}$$

#### AP Calculus

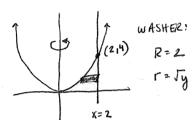
#### 7.3 Worksheet (day 3)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Let R be the region between the graphs of y = 1 and  $y = \sin x$  from x = 0 to  $x = \frac{\pi}{2}$ . What is the volume of the solid obtained by revolving R about the x-axis?



2. The region enclosed by the graph of  $y = x^2$ , the line x = 2, and the x-axis is revolved about the y-axis. What is the volume of the solid generated? solid generated?



 $=\pi\left[\left(4-\frac{1}{2}\cdot16\right)-\left(0-0\right)\right]=\boxed{8\pi}$ 

3. [Calculator] A region in the first quadrant is enclosed by the graphs of  $y = e^{2x}$ , x = 1, and the coordinate axes. If the region is rotated about the y-axis, what is the volume of the solid generated? rotated about the y-axis, what is the volume of the solid generated?

NEEDS 2 STR 1PS!

$$\frac{1}{2} \ln y = x$$

$$\frac{1}{$$

TOTAL = 
$$\pi \int_{0}^{e^{2}} \left[1 - \left(\frac{1}{2} \ln y\right)^{2}\right] dy + \pi \int_{0}^{1} dy$$

$$\approx 13.17749851$$

4. [Calculator] Let R be the region in the first quadrant enclosed by the graph of  $y = (x+1)^{1/3}$ , the line x = 7, the x-axis, and the yaxis. What is the volume of the solid generated when R is revolved about the y-axis?

NEEDS 2 STRIPS!

WASHER: 
$$R = 7$$
 $T = y^3 - 1$ 

DISC:  $R = 7$ 
 $T = y^3 - 1$ 
 $T = y^3 - 1$ 
 $T = y^3 - 1$ 

- 5. Let R be the region in the first quadrant enclosed by the graphs of y = 2x and  $y = x^2$ , as shown in the figure to the right.
  - a) Find the area of R.

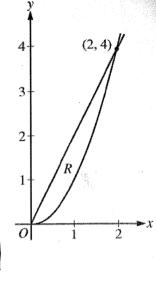
$$\left[\int_{0}^{2} (2x - x^{2}) dx\right] = x^{2} - \frac{x^{3}}{3}\Big|_{0}^{2} = (4 - \frac{8}{3}) - (0) = \frac{4}{3}$$

b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area  $A(x) = \sin(\frac{\pi}{2}x)$ . Find the volume of the solid.

$$\int_{0}^{2} A(x) dx = \int_{0}^{2} \sin (\pi/2x) dx = \frac{-\cos (\pi/2x)}{\pi/2} \Big|_{0}^{2}$$

$$= \frac{2}{\pi} \left[ -\cos (\pi/2\cdot 2) - (-\cos (\pi/2\cdot 0)) \right]$$

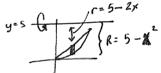
$$= \frac{4}{\pi}$$



c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

- (214) SIDE OF SQUARE = Jy 2y
- d) Write but do not evaluate, the integral which gives the volume of the solid formed by rotating R around the line y = 5.

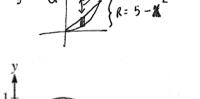
$$\int_{0}^{2} \left[ (5-x^{2})^{2} - (5-2x)^{2} \right] dx$$



- 6. [Calculator] Let R be the region bounded by the graphs of  $y = \sin(\pi x)$ and  $y = x^3 - 4x$ , as shown in the figure above.
  - a) Find the area of R.

$$\int_0^2 \left[ \sin \left( \pi x \right) - \left( x^3 - 4 x \right) \right] dx = 4 \text{ units}^2$$

b) The horizontal line y = -2 splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.



R

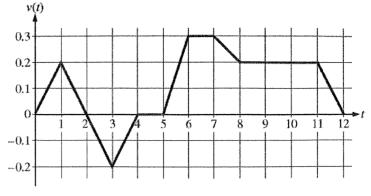
$$x^3-4x=-2$$
 at  $x=.53918887$  A  $4x=.6751309$  B

- $(-2) (x^3 4x)$  dx
- c) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by h(x) = 3 - x. Find the volume of water in the pond.

$$V = \int_{0}^{2} \left[ \sin(\pi x) - (x^{3} - 4x) \right] \cdot (3 - x) dx \approx \left[ 8.369953/06 \right]$$

$$\approx 8.369 \text{ or } 8.370$$

7. [Calculator] Caren rides her bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time t = 12 minutes. During the time interval  $0 \le t \le 12$  minutes, her velocity, v(t), in miles per minute, is modeled by the piecewise-linear function whose graph is shown at the right.



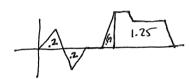
a) Find the acceleration of Caren's bicycle at time t = 7.5 minutes. Indicate units of measure.

$$a(7.5) = v'(7.5) = \frac{-1}{1} = \begin{bmatrix} -1 & \text{mile} \\ \text{min}^2 \end{bmatrix} -0.2$$
slope of velocity

cet  $t = 7.5$ 

b) Using correct units, explain the meaning of  $\int_{0}^{12} |v(t)| dt$  in terms of Caren's trip. Find the value of  $\int_{0}^{12} |v(t)| dt$ .

Solv(t) at is the total distance Caren traveled in the 12 minutes



c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

velocity indicates direction... since v(t) >0 on (0,2) € v(t) <0 on (2,4)

Caren Changed directions at t=2

She went . 2 miles toward school, then returned home, spent I minute at home, then went to school again

d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by  $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12}t\right)$ , where w(t) is in miles per minute for  $0 \le t \le 12$  minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

Careris Distance from school:

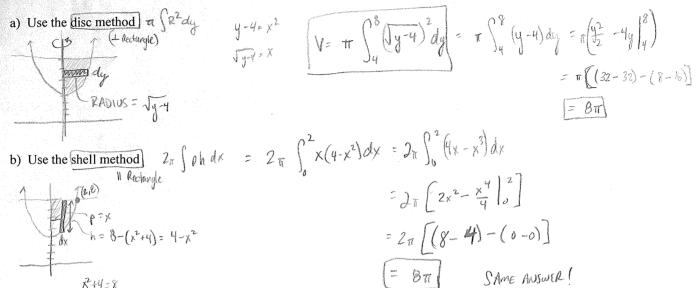
$$\int_{0}^{12} v(t)dt = .2 - .2 + .15 + 1.25 = [1.4 \text{ miles}]$$

LARRY'S Distance from school:

Caren lives closer

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Let R be the region bounded by the graphs of  $y = x^2 + 4$ , y = 8, and x = 0, set up and evaluate the integral that gives the volume of the solid generated by revolving R about the y - axis.



Questions 2 and 3 were done on the last worksheet using 2 rectangles and the washer method. Try doing them again using the shell method.

2. [Calculator] A region in the first quadrant is enclosed by the graphs of  $y = e^{2x}$ , x = 1, and the coordinate axes. If the region is rotated about the y-axis, what is the volume of the solid generated?

$$V = 2\pi \int_{0}^{1} x \cdot e^{2x} dx = 13.17749851$$

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$$V = 2\pi \int_{0}^{1} x \cdot e^{2x} dx = 13.17749851$$

3. [Calculator] Let R be the region in the first quadrant enclosed by the graph of  $y = (x+1)^{\frac{1}{3}}$ , the line x = 7, the x-axis, and the y-axis. What is the volume of the solid generated when R is revolved about the y-axis?

$$V = 2\pi \int_{0}^{7} x(x+1)^{3} dx = (271.2989656)$$

$$V = 2\pi \int_{0}^{7} x(x+1)^{3} dx = (271.2989656)$$

$$271.298 \text{ or } 271.299$$

$$|| \text{Reckingly}|| \Rightarrow \text{SHELL} \Rightarrow 2\pi \int_{0}^{7} p \, h \, dy$$

4. Find the volume of the solid formed by revolving the region bounded by the graphs of y = x and  $y = 4x - x^2$  about the y-axis.

$$\rho = x$$
 $h = (4x - x^2) - (x) = 3x - x^2$ 

$$V = 2\pi \int_{0}^{3} x (3x - x^{2}) dx = 2\pi \int_{0}^{3} (3x^{2} - x^{3}) dx$$

$$= 2\pi \left[ x^{3} - \frac{x^{4}}{4} \Big|_{0}^{3} \right]$$

$$= 2\pi \left[ (27 - 8/4) - (0 - 0) \right]$$

$$= 2\pi \left[ \frac{108 - 81}{4} - \frac{127\pi}{4} \right]$$

5. The shaded region R, shown in the figure below, is rotated about the y-axis to form a solid who volume is 10 cubic inches. Of the following, which best approximates k?

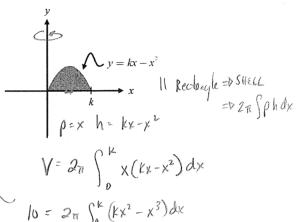
D) 4.18

$$10 = 2\pi \left[ \frac{\kappa \cdot \frac{\chi^{3}}{3} - \frac{\chi^{4}}{4} \Big|_{0}^{\kappa} \right]$$

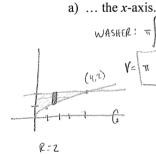
$$10 = 2\pi \left[ \left( \frac{k^4}{3} - \frac{k^4}{4} \right) - \left( 0 - 0 \right)^{\frac{1}{4}} \right]$$

$$lo = I\pi \left( \frac{4k^4 - 3k^4}{17} \right)$$

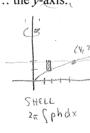
E) 
$$4.77$$
  $10 = \frac{\pi \cdot k}{6}$ 

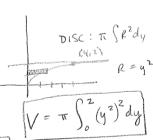


6. Set up an integral and use your calculator to find the volume of the solid generated by revolving the region bounded by  $y = \sqrt{x}$ and the lines y = 2 and x = 0 about ... (if possible, set up the problem two different ways).

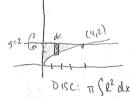


$$\frac{1}{1} \frac{1}{1} \frac{1}$$



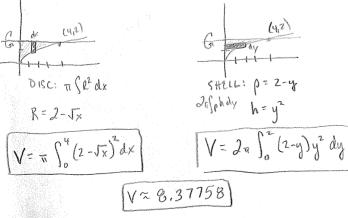


$$h = 2 - \sqrt{x}$$
  
the line  $r = 4$ 

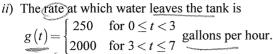


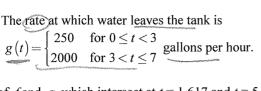
$$V = 2\pi \int_0^4 (4-x)(2-5x)dx$$

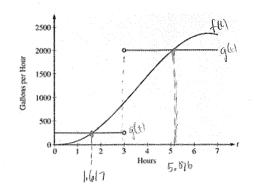
WASHER: 
$$R = 4$$
,  $r = 4-y^2$   
 $\pi \int (k^2 - r^2) dy$ 



- 7. [Calculator] The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \le t \le 7$ , where t is measured in hours. In this model, rates are given as follows:
  - i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \le t \le 7$ .







The graphs of f and g, which intersect at t = 1.617 and t = 5.076, are shown in the figure to the right. At time t = 0, the amount of water in the tank is 5000 gallons.

a) How many gallons of water enter) the tank during the time interval  $0 \le t \le 7$ ? Round your answer to the nearest gallon.

b) For  $0 \le t \le 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for your answer.

c) For  $0 \le t \le 7$ , at what time t is the amount of water in the tank the greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

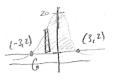
Find maximums at endpoints 
$$(t=0 \text{ or } t=7)$$

where derivative = 0 ... It the derivative is just the rate of change coff water in the part of water in the solution of where  $t=0$  is the solution of where  $t=0$  is the solution of  $t=0$  in the part of water is shown of  $t=0$  in the pump of  $t=0$  in th

I. the max amount of water in the took is = 5127 gallons @ t=3

- 8. [Calculator] Let R be the region in the first and second quadrants bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line y = 2.
  - a) Find the area of R.

$$A = \int_{-3}^{3} \left( \frac{20}{1+x^2} - 2 \right) dx \approx \left[ 37.9613309 \right]$$
37.961 or 37.96



$$\frac{20}{1+y^2} = 2$$

$$20 = 2 + 2x$$

$$18 = 2x^2$$

$$9 = x^2$$

b) Find the volume of the solid generated when R is rotated about the x-axis.

waster ... 
$$R = \frac{20}{1+x^2}$$
 ...  $r = 2$ 

$$\pi \int (e^2 - r^2) dx$$

$$V = \pi \int_{-3}^{3} \left( \frac{20}{1+\chi^2} \right)^2 - (2)^2 dx$$

$$V = 1871.190104$$

c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

DIAMETER = 
$$\frac{20}{1+x^2} - 2$$

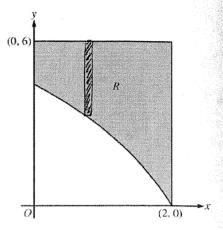
PADIUS =  $\left[\frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)\right] = \frac{10}{1+x^2} - 1$ 

AREA OF SEMICIRCLE = 
$$\frac{1}{2}\pi \left(\frac{10}{1+x^2}-1\right)^2$$

TOTAL = 
$$\int_{-3}^{3} \frac{\pi}{2} \left( \frac{10}{1+x^2} - 1 \right)^2 dx \approx \left[ \frac{174}{2}, \frac{2684584}{3} \right]$$

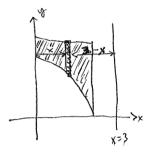
- 1. [Calculator] In the figure above, R is the shaded region in the first quadrant bounded by the graph of  $y = 4\ln(3 - x)$ , the horizontal line y = 6, and the vertical line x = 2.
- a) Find the volume of the solid generated when R is revolved about the line y = 8.

$$l = 8 - 4 \ln(3 - x)$$
 $T = 8 - 4 \ln(3 - x)^2 - (2)^2 dx \approx 2$ 



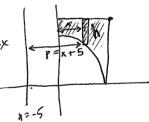
b) Find the volume of the solid generated when R is revolved about the line x = 3.

$$2\pi \int_{0}^{2} (3-x) (6-4\ln(3-x)) dx$$



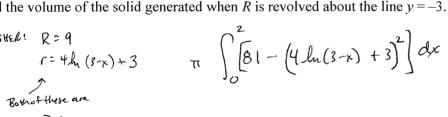
c) Find the volume of the solid generated when R is revolved about the line x = -5.

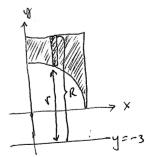
$$2\pi \int_{0}^{2} (5+x) (6-4\ln(3-x)) dx$$



d) Find the volume of the solid generated when R is revolved about the line y = -3.

$$\int \left[ 81 - \left( 4 \ln(3-x) + 3 \right)^{2} \right] dx$$





- 2. Complete the following questions from your textbook:
  - (7.) Page 386 #8, 10, 12, and 14.
  - (7.3) Pages 407 409 #14, 30, 34, 45, 46, 47 and 49
  - Rev Pages 430 433 #3 7, 10, 13, 21, 22, 25, 39, and 44