AP Calculus
7.3 Worksheet (day 3)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. What is the volume of the solid obtained by revolving R about the x-axis?

2. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. What is the volume of the solid generated?

3. [Calculator] A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, what is the volume of the solid generated?

4. [Calculator] Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the x-axis, and the y-axis. What is the volume of the solid generated when R is revolved about the y-axis?

5. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure to the right.

   a) Find the area of R.

   b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
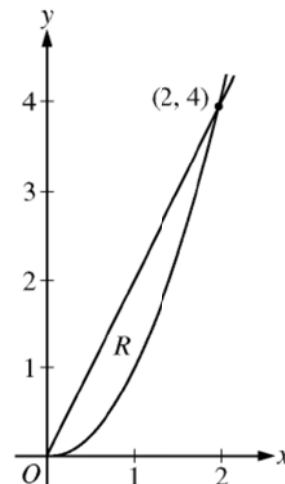
   c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

   d) Write but do not evaluate, the integral which gives the volume of the solid formed by rotating R around the line $y = 5$.

6. [Calculator] Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   a) Find the area of R.

   b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
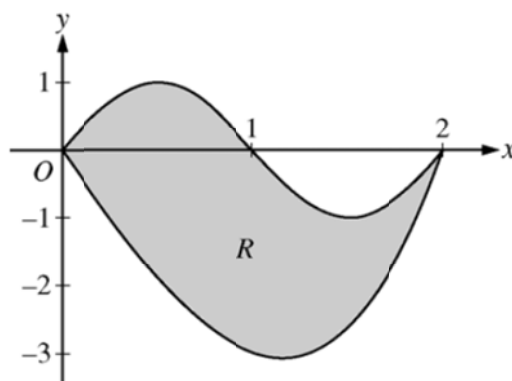
   c) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

7. [Calculator] Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity, $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown at the right.
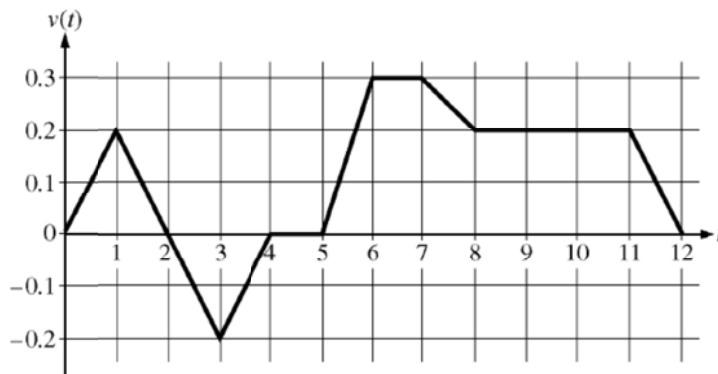
a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

b) Using correct units, explain the meaning of $\int_0^{12} |v(t)| \, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)| \, dt$.

c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12} t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.