AP Calculus
5.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

FTOC: The Evaluation Part

1. Complete the following questions from your textbook: Page 303 # 27, 29, 30, 35, 37 – 40, 43, 45

FTOC: The Derivative of an Integral Part (Simple)

2. Complete the following questions from your textbook: Page 302 #2, 3, 4, 6

FTOC: The Derivative of an Integral Part (Extended)

3. Find $\frac{d}{dx}\left[\int_{1}^{\sin x} \sqrt{1+t^3}\ dt\right]$.

4. Find $\frac{d}{dx}\left[\int_{\sin x}^{x^3} f\left(t\right) dt\right]$

5. Find $\frac{d}{dx}\left[\int_{\sin x}^{x^3} e^{t^2}\ dt\right]$

6. Complete the following questions from your textbook: Page 302 #11, 17, 28

Putting it all together …

7. Complete the following questions from your textbook: Page 303 #57, 59

8. What are all the values of *k* for which $\int_{2}^{k} x^2\ dx = 0$?

A –2

B 0

C 2

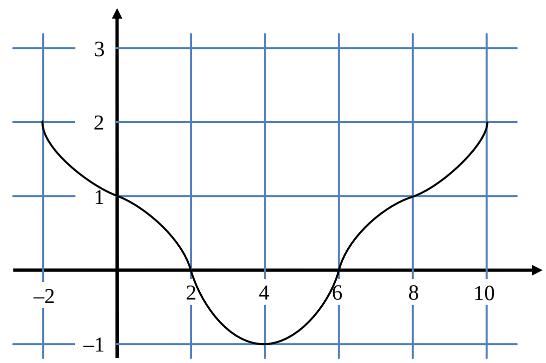
D –2 and 2

E –2, 0, and 2

9. The graph of a differentiable function f on the interval [–2, 10] is shown in the figure below. The graph of f has a horizontal tangent line at $x = 4$. Let $h(x) = 9 + \int_4^x f(t)\,dt$ for $-2 < x < 10$.

a) Find $h(4)$, $h'(4)$, and $h''(4)$

Graph of *f*



b) On what intervals is h increasing? Justify your answer.

c) On what intervals is h concave downward? Justify your answer.

d) Find the Trapezoidal Sum to approximate $\int_{-2}^{10} f(x)\,dx$ using 6 subintervals of length = 2.

10. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table below gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (The Volume of a sphere is given by $V = \frac{4}{3}\pi r^3$)

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

c) Use a right Riemann Sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

d) Is your approximation is part c greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.