AP Calculus
4.5 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Consider the function $y = \sin x$.

a) Find the equation of the tangent line when $x = 0$.

b) Graph both equations on your calculator in a standard viewing window. Is the tangent line a good approximation for the curve? Zoom in (at the origin) a couple of times. What do you notice?

c) Use the tangent line to approximate sin (0.2).

2. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

A 2.00

B 2.03

C 2.06

D 2.12

E 2.24

3. [With calculator] Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate the values of $f(x)$. Which of the following is the greatest value for which the error resulting from this tangent line approximation is less than 0.5?

A 2.4

B 2.5

C 2.6

D 2.7

E 2.8

4. Find the differential dy when $dx = -0.2$ and $x = 1$, if $y = x^2 e^x$. Explain what you've found.

5. Without a calculator, use differentials to approximate $\sqrt[4]{19}$.

6. [Calculator Required] The range *R* of a projectile is $R = \frac{v_0^{\,2}}{32}(\sin 2\theta)$, where $v_0^{\,2}$ is the initial velocity in feet per second and θ is the angle of elevation. Let $v_0 = 2200$ feet per second and let θ change from 10° to 11°.
[Remember … in calculus, we never use degrees!]

a) Find the actual change in the range.

b) Use differentials to *approximate* the change in the range. Are your answers "close" ?

7. Use linearization to approximate $f(0.1)$ if $f(x) = \frac{1}{\sqrt{4+x}}$. Find the error for your approximation.

8. If $y = \sin\left(x^2 - 3\right)$, find *dy* if $x = \sqrt{3}$ and $dx = \frac{1}{10}$.

9. The radius of a ball bearing is measured to be 0.7 inch. If the measurement is correct to within 0.01 inch, estimate the error in the volume of the ball bearing. [$V = \frac{4}{3}\pi r^3$]

10. How accurately should you measure the radius of a sphere in order to be reasonably sure the volume of the sphere is within 2% of its actual value? [Volume of a sphere: $V = \frac{4}{3}\pi r^3$]

11. A right circular cone has a radius that is one-third of the height. How accurately must the radius be measured so that error in calculating the volume is no more than 3% ? [Volume of a cone: $V = \frac{1}{3}\pi r^2 h$]

12. The circumference of the equator of a sphere is measured as 10 cm with a possible error of 0.4 cm. This measurement is then used to calculate the radius. The radius is then used to calculate the surface area and volume of the sphere. Estimate the percentage errors in the calculated values of the following:

a) the radius

b) the surface area [$S = 4\pi r^2$]

c) the volume