

AP Calculus  
4.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Circle the correct word that completes the following statements:
  - a) When the graph of  $f$  is increasing,  $f'$  is ( positive , negative ).
  - b) When the graph of  $f$  is decreasing,  $f'$  is ( positive , negative ).
  - c) The graph of  $f$  is concave upward when  $f''$  is ( positive , negative ).
  - d) The graph of  $f$  is concave downward when  $f''$  is ( positive , negative ).
  - e) The graph of  $f$  is concave upward when  $f'$  is ( increasing , decreasing ).
  - f) The graph of  $f$  is concave downward when  $f'$  is ( increasing , decreasing ).
  
2. Use the function  $y = 3x - x^3 + 5$ . [No calculator allowed]
  - a) Where is the function increasing? Justify your response.
  
  - b) Where is the function decreasing? Justify your response.
  
  - c) Where is the function concave up? Justify your response.
  
  - d) Where is the function concave down? Justify your response.
  
  - e) Where are the point(s) of inflection? Justify your response.
  
  - f) Find ALL extrema and justify your response.
  
  - g) Create a *sketch* of the function.

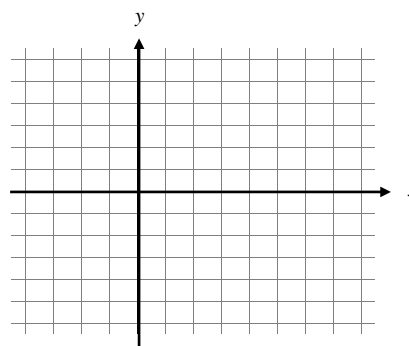
3. [2005 AP Calculus AB Free Response #4 ... No Calculator Allowed]

| $x$      | 0  | $0 < x < 1$ | 1 | $1 < x < 2$ | 2   | $2 < x < 3$ | 3  | $3 < x < 4$ |
|----------|----|-------------|---|-------------|-----|-------------|----|-------------|
| $f(x)$   | -1 | Negative    | 0 | Positive    | 2   | Positive    | 0  | Negative    |
| $f'(x)$  | 4  | Positive    | 0 | Positive    | DNE | Negative    | -3 | Negative    |
| $f''(x)$ | -2 | Negative    | 0 | Positive    | DNE | Negative    | 0  | Positive    |

Let  $f$  be a function that is continuous on the interval  $[0, 4]$ . The function  $f$  is twice differentiable except at  $x = 2$ . The function  $f$  and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of  $f$  do not exist at  $x = 2$ .

a) For  $0 < x < 4$ , find all values of  $x$  at which  $f$  has a relative extremum. Determine whether  $f$  has a relative maximum or a relative minimum at each of these values. Justify your answer.

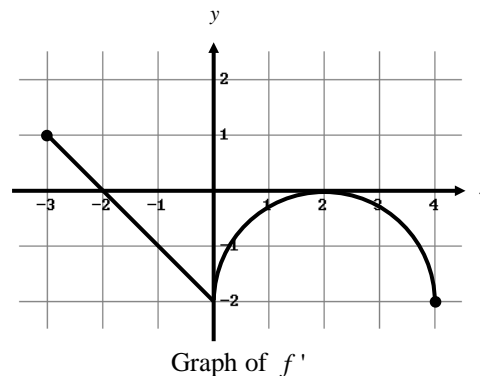
b) On the axes provided, sketch the graph of a function that has all the characteristics of  $f$ .



4. [2003 AP Calculus AB Free Response #4 ... No Calculator Allowed] Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown below.

a) On what intervals, if any, is  $f$  increasing? Justify your answer.

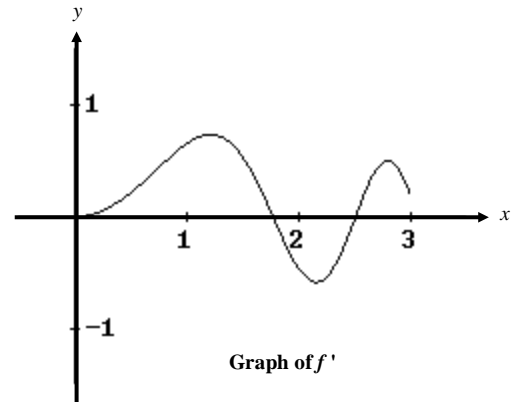
b) Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.



c) Find an equation for the line tangent to the graph of  $f$  at the point  $(0, 3)$ .

5. [2006 AP Calculus AB Form B Free Response #2 ... Calculator Required] Let  $f$  be a function defined for  $x \geq 0$  with  $f(0) = 5$  and  $f'$ , the first derivative of  $f$ , given by  $f'(x) = e^{(-x/4)} \sin(x^2)$ . The graph of  $y = f'(x)$  is shown below.

a) Use the graph of  $f'$  to determine whether the graph of  $f$  is concave up, concave down, or neither on the interval  $1.7 < x < 1.9$ . Explain your reasoning.



b) Write an equation for the line tangent to the graph of  $f$  at  $x = 2$ .

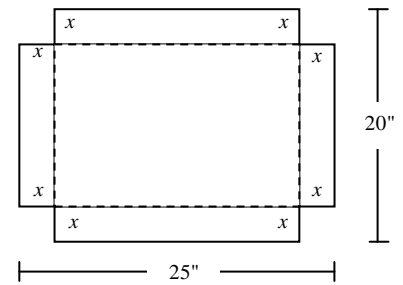
6. [2004 AP Calculus AB Free Response Question #4 ... No Calculator Allowed] Consider the curve given by  $x^2 + 4y^2 = 7 + 3xy$ .

a) Show  $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$

b) Show that there is a point  $P$  with  $x$  - coordinate 3 at which the line tangent to the curve  $P$  is horizontal. Find the  $y$  - coordinate of  $P$ .

c) Find the value of  $\frac{d^2y}{dx^2}$  at the point  $P$  found in part  $b$ . Does the curve have a local maximum, a local minimum, or neither at the point  $P$ ? Justify your answer.

7. An open-top box is to be made by cutting congruent squares of side length  $x$  from the corners of a 20- by 25-inch sheet of tin and bending up the sides (see figure below). How large should the squares be to make the box hold as much as possible? What is the resulting volume?



8. If  $g$  is a differentiable function such that  $g(x) < 0$  for all real numbers  $x$ , and if  $f'(x) = (x^2 - 9)g(x)$ , which of the following is true?

- A)  $f$  has a relative maximum at  $x = -3$  and a relative minimum at  $x = 3$ .
- B)  $f$  has a relative minimum at  $x = -3$  and a relative maximum at  $x = 3$ .
- C)  $f$  has relative minima at  $x = -3$  and at  $x = 3$ .
- D)  $f$  has relative maxima at  $x = -3$  and at  $x = 3$ .
- E) It cannot be determined if  $f$  has any relative extrema.

9. Complete the following questions from the textbook:

- 4.3: p215 – 217 #1, 4, 7, 9, 13, 20, 27, 35, 39, 49, 51,
- 4.4 p228 – 231 #30, 40, 56

You should also start reviewing for your chapter 3.6 – 4.4 exam by completing the following questions from the textbook:

- Page 181 #5 – 11, 32 – 48 even, 51, 55, 62, 67, 68, 70
- Page 256 #1, 3, 5, 17, 21, 23, 25, 26, 27, 30, 31, 32, 33, 35, 36, 37, 40, 45, 49, 52