SEPARATE AND INTEGRATE (§6.1 & §6.4)

A method of solving differential equations … move all y – values to the left side and all x – values to the right side including the dx (*separate*) and the take the antiderivative of both sides (*integrate*).

To completely solve the differential equation, you need to know some initial conditions so you can find the constant of integration, C. and you need to solve for y.

Example: Solve the differential equation $\frac{dy}{dx} = (1 + \ln x)\, y$ if $y = 1$ when $x = 1$.

1st: **Separate** the variables … without this step, you get 0 credit!

$$\frac{1}{y}\, dy = (1 + \ln x)\, dx$$

2nd: **Integrate** both sides

$$\int \frac{1}{y}\,dy = \int (1+\ln x)\,dx$$

$$\ln|y| = x + [x\ln x - x] + C$$

$$\ln|y| = x\ln x + C$$

♪: $\int \ln x\,dx$ is done with integration by parts … see notecard on Special Cases of Lipet.

3rd: Solve for *y*

$y = e^{x\ln x + C} = e^{x\ln x}e^{C} = De^{x\ln x}$, where $D = e^{C}$, just another constant.

4th: Solve for the constant, *D*, using the initial conditions … *y* = 1 when *x* = 1.

$$1 = De^{1\cdot\ln(1)}$$

$$1 = De^{0}$$

$$1 = D$$

Therefore, the solution to the differential equation is $y = e^{x\ln x}$.