LOGARITHMIC DIFFERENTIATION (§3.9)

Using the rules of logarithms to rewrite a function so that you can find the derivative.

Example: If $f(x) = \left(x^2 + 1\right)^{(2-3x)}$, then $f'(1) =$

A $-\frac{1}{2}\ln(8e)$

B $-\ln(8e)$

C $-\frac{3}{2}\ln(2)$

D $-\frac{1}{2}$

E $\frac{1}{8}$

First, take the natural log of both sides and rewrite the right side using log rules to obtain

$$\ln(y) = \ln\left(x^2+1\right)^{(2-3x)}$$

$$\ln(y) = (2-3x)\cdot\ln\left(x^2+1\right)$$

Next, use implicit differentiation (don't forget to use the product rule on the right) to get

$$\frac{1}{y}\cdot\frac{dy}{dx} = (2-3x)\cdot\frac{2x}{x^2+1} + \ln\left(x^2+1\right)(-3)$$

Solving for $\frac{dy}{dx}$ gives you

$$\frac{dy}{dx} = \left[(2-3x)\cdot\frac{2x}{x^2+1} + \ln\left(x^2+1\right)(-3)\right]\cdot y$$

Since $y = \left(x^2+1\right)^{(2-3x)}$, we have

$$f\,'(x) = \left[(2-3x)\cdot\frac{2x}{x^2+1} - 3\ln\left(x^2+1\right)\right]\cdot\left(x^2+1\right)^{(2-3x)}$$

This concludes the logarithmic differentiation steps.

To find the derivative at x = 1, we plug in x = 1

$$f'(1) = \left[(2-3\cdot 1)\cdot\frac{2\cdot 1}{1^2+1} + \ln\left(1^2+1\right)(-3)\right]\cdot\left(1^2+1\right)^{(2-3\cdot 1)}$$

$$= \left[-1-3\ln(2)\right](2)^{-1}$$

For a free response question, you would be done … but we have choices …
To make our answer look like one of the choices, substitute 1 = ln e, factor out a negative and write 2^{-1} as $\frac{1}{2}$. This makes

$$f'(1) = -\left[\ln e + 3\ln(2)\right]\tfrac{1}{2}$$

Since $3\ln(2) = \ln 2^3 = \ln 8$ we have

$$f'(1) = -\left[\ln e + \ln 8\right]\tfrac{1}{2}$$

Since ln e + ln 8 = ln (8e) can now write

$$f'(1) = -\tfrac{1}{2}\ln(8e)$$

Thus, the answer is A.