INTEGRAL OF TRIG^2 FUNCTIONS (§6.3)

$$\int \sin^2 x \, dx = \tfrac{1}{2}x - \tfrac{1}{4}\sin(2x) + C$$

$$\int \cos^2 x \, dx = \tfrac{1}{2}x + \tfrac{1}{4}\sin(2x) + C$$

$$\int \tan^2 x \, dx = \tan x - x + C$$

$$\int \cot^2 x \, dx = -\cot x - x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\csc^2 x + C$$

Proofs …

$\int \sec^2 x\, dx = \tan x + C$ and $\int \csc^2 x\, dx = -\cot x + C$ should not require a proof.

The proofs of the other four require the following trig indentities …

$$\sin^2 x = \frac{1-\cos(2x)}{2} = \tfrac{1}{2} - \tfrac{1}{2}\cos(2x)$$

$$\cos^2 x = \frac{1+\cos(2x)}{2} = \tfrac{1}{2} + \tfrac{1}{2}\cos(2x)$$

$$\tan^2 x = \sec^2 x - 1$$

$$\cot^2 x = \csc^2 x - 1$$

$$\int \sin^2 x\, dx = \int \tfrac{1}{2} - \tfrac{1}{2}\cos(2x) = \tfrac{1}{2}x - \tfrac{1}{4}\sin(2x) + C$$

$$\int \cos^2 x\, dx = \int \tfrac{1}{2} + \tfrac{1}{2}\cos(2x) = \tfrac{1}{2}x + \tfrac{1}{4}\sin(2x) + C$$

$$\int \tan^2 x\, dx = \int \sec^2 x - 1 = \tan x - x + C$$

$$\int \cot^2 x\, dx = \int \csc^2 x - 1\, dx = -\cot x - x + C$$