DERIVATION OF AN EXPONENTIAL FUNCTION (§6.4)

Suppose y is a function of time t.
When the rate of change of a variable y is proportional to the value of y we say

$$\frac{dy}{dt} = k \cdot y ,$$

where k is the constant of proportionality.

Use the initial condition $y = y_0$ when $t = 0$.

Solve this differential equation AND write the answer as a function of x by *separating* the variables and *integrating*. First step, get y on the left and dt on the right.

$$\frac{dy}{y} = k \cdot dt$$

Next, integrate both sides. $\int \frac{dy}{y} = \int k \cdot dt$

$$\ln|y| = kt + C$$

Rewriting using rules of logarithms we get

$$y = e^{kt+C}$$

(Since $e^{\text{any power}}$ is always positive, we don't need the absolute value of y anymore).

Using rules of exponents gives us $y = e^{kt}e^{C}$.

Since e^{C} is just a constant, we will replace it with D, another constant, to obtain

$$y = De^{kt}$$

Now, use the initial condition that when $t = 0$, $y = y_0$ to get

$$y_0 = De^{k(0)}$$

$$y_0 = D$$

Thus, the solution to the original differential equation with the given initial condition is

$$y = y_0 e^{kt}$$