INTRODUCTION TO CONICS

While there are many related ideas and topics that can be used with conics, we are only attempting to scratch the surface. We are only interested in your ability to identify/classify conics by their equation and their graph as well as write an equation of the following conic sections: Circles, Ellipses, and Hyperbolas … you should add these to your parent functions list.

First, what is a conic section? A conic section is the cross-sectional shape formed by slicing a double cone (a cone on top of another cone). Again, we are going to deal with 3 of these … circles, ellipses, and hyperbolas.

Got to the following website for an animated view of these conic sections:
http://mathdemos.gcsu.edu/mathdemos/family_of_functions/conic_gallery.html

General Form of a Conic Section

All conics can be written in the form $Ax^2 + By^2 + Cx + Dy + Exy + F = 0$.
The conics we will be focusing on will have no *xy* term (unless that is the only term). If $E \neq 0$, the conic will be rotated.

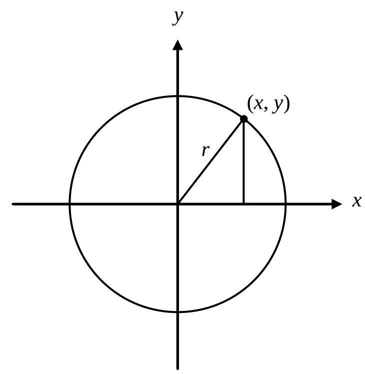
Circles

Consider a circle centered at the origin, with radius *r*. The Pythagorean Theorem, allows us to generate an equation of the circle to be

$$x^2 + y^2 = r^2 .$$

If we divide both sides of the equation by r^2, we get

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$



To graph the circle yourself, find the center, and plot 1 point that is *r* spaces up, down, left and right of the center.

Example 1: Graph each of the following equations:

a) $x^2 + y^2 = 36$

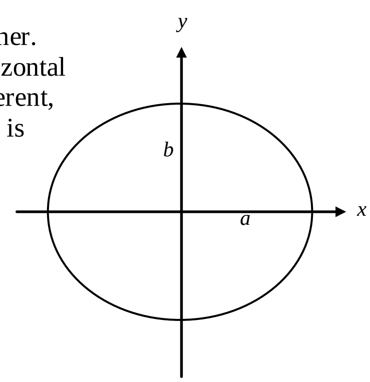
b) $(x-3)^2 + (y+2)^2 = 25$

Ellipses

An ellipse is simply a circle that has been stretched more in one direction than the other. That "stretch" can be seen in the equation of a circle with the r^2, where both the horizontal stretch and vertical stretch are *r* spaces. If the horizontal and vertical stretch are different, the values of each denominator will be different. Thus the general form of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 ,$$

where *a* is the horizontal stretch from the center, and
b is the vertical stretch from the center.



To graph the ellipse yourself, find the center, and plot 1 point that is *a* spaces left and right of the center, and 1 point that is *b* spaces up and down from the center.

The mathematical definition of an ellipse is the set of all points whose distances from two fixed points (called foci) have a constant sum. Light (or sound) that originates at one focus point inside of the ellipse will reflect off the ellipse to the other focus point.

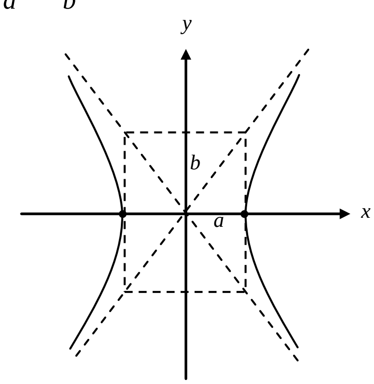
Example 2: Graph each of the following equations:

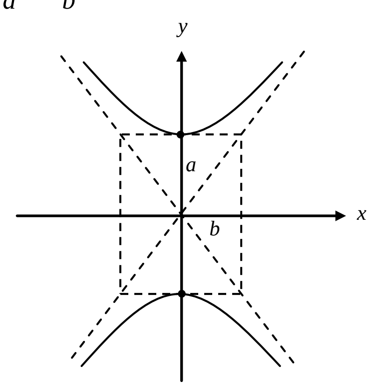
a) $\dfrac{x^2}{25}+\dfrac{y^2}{36}=1$ b) $\dfrac{(y-1)^2}{16}+\dfrac{(x+3)^2}{4}=1$

Hyperbolas

The mathematical definition of a hyperbola is the set of all point whose distances from two fixed points (called foci) have a constant difference. Thus, the equation for a hyperbola is very similar to an ellipse, except you are subtracting instead of adding. Since the order in which you subtract matters, the hyperbola opens in the direction of whichever axis comes first.

$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ … opens left and right OR $\dfrac{y^2}{a^2}-\dfrac{x^2}{b^2}=1$ … opens up and down





To graph a hyperbola yourself, find the "center", create a rectangular box. Whatever the square root of the denominator is below *x*, that is the distance you move left and right of the "center". Similarly, the square root of the denominator below *y* dictates how far you move up and down from the "center". A hyperbola has slanted asymptotes that go through the corners of your rectangular box. If *x* comes first (meaning you are subtracting *y*), your hyperbola will open left and right, but if *y* comes first then your hyperbola will open up and down. The only "points" you need to show will be the "vertices" which are actually on the rectangular box.

For applications of these conic sections or more examples, see Appendix Section A5 in your textbook starting on page 578.

Example 3: Graph each of the following equations:

a) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

b) $(y-4)^2 - \frac{(x+1)^2}{16} = 1$