AP Calculus AB
Definite Integrals (U-sub)

Name: KEY
Block: ________

1. $\int_{1}^{\sqrt{2}} x \cdot 2^{-x^2}\,dx$

$\boxed{\frac{1}{8\ln 2}}$

2. $\int_{0}^{2} \sqrt{4x+1}\,dx$

$\boxed{\frac{13}{3}}$

3. $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

$\boxed{\pi/2}$

4. $\int_{0}^{1} \frac{1}{(2x+3)^3}\,dx$

$\boxed{\frac{4}{225}}$

5. $\int_{e}^{e^2} \frac{1}{x\ln x}\,dx$

$\boxed{\ln 2}$

6. $\int_{2}^{e+1} \frac{x}{(x-1)^2}\,dx$

$\boxed{2-\frac{1}{e}}$

7. $\int_{0}^{\pi} \sin\left(\frac{x}{2}\right)dx$

$\boxed{2}$

8. $\int_{-\pi}^{\pi} x\sin\left(x^2\right)dx$

$\boxed{0}$

9. $\int_{-1}^{0} \frac{2}{6x-1}\,dx$

$\boxed{-\frac{1}{3}\ln 7}$

10. $\int_{1}^{5} \frac{(\ln x)^{1/2}}{x}\,dx$

$\boxed{\frac{2}{3}[\ln(5)]^{3/2}}$

11. $\int_{0}^{1} xe^{-x^2}\,dx$

$\boxed{\frac{-1+e}{2e}}$

12. $\int_{0}^{2} \left(2^x + x^2\right)dx$

$\boxed{\frac{3}{\ln 2} + \frac{8}{3}}$

① $\int_1^{\sqrt{2}} x \cdot 2^{-x^2}\, dx$

Let $u = -x^2$

$du = -2x\, dx \Rightarrow -\frac{1}{2} du = x\, dx$

if $x = \sqrt{2}$, $u = -(\sqrt{2})^2 = -2$

if $x = 1$, $u = -(1)^2 = -1$

$$= -\frac{1}{2}\int_{-1}^{-2} 2^u\, du$$

$$= -\frac{1}{2}\left[\frac{1}{\ln 2} \cdot 2^u \Big|_{-1}^{-2}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{\ln 2} \cdot 2^{-2} - \frac{1}{\ln 2} \cdot 2^{-1}\right]$$

$$\boxed{= -\frac{1}{2}\left[\frac{1}{4\ln 2} - \frac{1}{2\ln 2}\right]}$$

$$-\frac{1}{2}\left(\frac{1-2}{4\ln 2}\right) = -\frac{1}{2}\boxed{\left(\frac{-1}{4\ln 2}\right)} = \boxed{\frac{+1}{\ln(256)}}$$

② $\int_0^2 \sqrt{4x+1}\, dx$

Let $u = 4x+1$

$du = 4\, dx \Rightarrow \frac{1}{4} du = dx$

if $x = 2$, $u = 4(2)+1 = 9$

if $x = 0$, $u = 4(0)+1 = 1$

$$= \frac{1}{4}\int_1^9 u^{1/2}\, du$$

$$= \frac{1}{4}\left(\frac{2}{3}u^{3/2}\Big|_1^9\right)$$

$$= \frac{1}{6}(9)^{3/2} - \frac{1}{6}(1)^{3/2}$$

$$= \frac{9}{2} - \frac{1}{6}$$

$$= \frac{26}{6} = \boxed{\frac{13}{3}}$$

③ $\int_{-1}^{1} \frac{1}{1+x^2}\, dx = \tan^{-1}(x)\Big|_{-1}^{1}$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \pi/4 - (-\pi/4)$$

$$= 2\pi/4$$

$$= \boxed{\pi/2}$$

④ $\int_0^1 \frac{1}{(2x+3)^3}\,dx$

Let $u = 2x+3$

$du = 2dx \Rightarrow \frac{1}{2}du = dx$

if $x=1$, $u = 2(1)+3 = 5$

if $x=0$, $u = 2(0)+3 = 3$

$= \frac{1}{2}\int_3^5 \frac{1}{u^3}\,du$

$= \frac{1}{2}\int_3^5 u^{-3}\,du$

$= \frac{1}{2}\left(-\frac{1}{2}u^{-2}\right)\Big|_3^5$

$= -\frac{1}{4u^2}\Big|_3^5$

$= -\frac{1}{4(5)^2} - \left(-\frac{1}{4(3)^2}\right)$

$= -\frac{1}{100} + \frac{1}{36}$

$= \frac{-9}{900} + \frac{25}{900}$

$= \frac{16}{900} = \boxed{\frac{4}{225}}$

⑤ $\int_e^{e^2} \frac{1}{x\ln x}\,dx$

Let $u = \ln x$

$du = \frac{1}{x}dx$

if $x = e^2$, $u = \ln(e^2) = 2$

if $x = e$, $u = \ln(e) = 1$

$= \int_1^2 \frac{1}{u}\,du$

$= \ln|u| \Big|_1^2$

$= \ln(2) - \ln(1)$

$\boxed{= \ln(2)}$

⑥ $\int_2^{e+1} \frac{x}{(x-1)^2}\,dx$

Let $u = x-1 \Rightarrow u+1 = x$

$du = dx$

if $\cancel{x} = e+1$, $u = e+1-1 = e$

if $x = 2$, $u = 2-1 = 1$

$= \int_1^e \frac{u+1}{u^2}\,du$

$= \int_1^e \left(\frac{u}{u^2} + \frac{1}{u^2}\right)du$

$= \int_1^e \left(\frac{1}{u} + u^{-2}\right)du$

$= \ln|u| - u^{-1}\Big|_1^e$

$\left(\ln(e) - \frac{1}{e}\right) - \left(\ln(1) - \frac{1}{1}\right)$

$= 1 - \frac{1}{e} + 1$

$\boxed{= 2 - \frac{1}{e}}$

(7) $\int_0^{\pi} \sin\left(\frac{x}{2}\right)dx = \int_0^{\pi} \sin\left(\tfrac{1}{2}x\right)dx = -\frac{\cos\left(\frac{1}{2}x\right)}{1/2}\Big|_0^{\pi}$

$= -2\cos\left(\tfrac{1}{2}x\right)\Big|_0^{\pi}$

$= \left[-2\cos(\pi/2)\right] - \left[-2\cos(0)\right]$

$= 0 + 2\cdot 1$

$\boxed{= 2}$

(8) $\int_{-\pi}^{\pi} x\sin(x^2)dx$

Let $u = x^2$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

if $\cancel{x} = -\pi$, $u = \pi^2$

if $x = \pi$, $u = \pi^2$

$\frac{1}{2}\int_{\pi^2}^{\pi^2} \sin(u)\,du = \boxed{0}$

Same upper & lower Limit!

(9) $\int_{-1}^{0} \frac{2}{6x-1}\,dx$

Let $u = 6x-1$

$du = 6\,dx \Rightarrow \frac{1}{6}du = dx$

if $x = 0$, $u = 6(0)-1 = -1$

if $x = -1$, $u = 6(-1)-1 = -7$

$\frac{1}{6}\int_{-7}^{-1} \frac{2}{u}\,du = \frac{1}{3}\int_{-7}^{-1} \frac{1}{u}\,du$

$= \frac{1}{3}\ln|u|\Big|_{-7}^{-1}$

$= \frac{1}{3}\ln|-1| - \frac{1}{3}\ln|-7|$

$= 0 - \frac{1}{3}\ln(7)$

$= -\frac{1}{3}\ln(7)$

(10) $$\int_1^5 \frac{(\ln x)^{1/2}}{x}\,dx$$

Let $u = \ln x$

$du = \frac{1}{x}\,dx$

if $x = 5$, $u = \ln 5$

if $x = 1$, $u = \ln 1 = 0$

$$= \int_0^{\ln 5} u^{1/2}\,du$$

$$= \left(\tfrac{2}{3}u^{3/2}\right)\Big|_0^{\ln 5}$$

$$= \tfrac{2}{3}(\ln 5)^{3/2} - \tfrac{2}{3}(0)^{3/2}$$

$$\boxed{= \tfrac{2}{3}(\ln 5)^{3/2}}$$

(11) $$\int_0^1 x e^{-x^2}\,dx$$

Let $u = -x^2$

$du = -2x\,dx \Rightarrow -\tfrac{1}{2}du = x\,dx$

if $x = 1$, $u = -(1)^2 = -1$

if $x = 0$, $u = -(0)^2 = 0$

$$= -\tfrac{1}{2}\int_0^{-1} e^u\,du$$

$$= -\tfrac{1}{2}e^u\Big|_0^{-1}$$

$$= -\tfrac{1}{2}(e^{-1}) - \left[-\tfrac{1}{2}(e^0)\right]$$

$$\boxed{= -\frac{1}{2e} + \tfrac{1}{2}}$$

$$= \boxed{\frac{-1+e}{2e}}$$

(12) $$\int_0^2 (2^x + x^2)\,dx = \frac{1}{\ln 2}\cdot 2^x + \frac{1}{3}x^3\Big|_0^2$$

$$= \left[\frac{1}{\ln 2}\cdot 2^2 + \frac{1}{3}(2)^3\right] - \left[\frac{1}{\ln 2}\cdot 2^0 + \frac{1}{3}\cdot(0)^3\right]$$

$$= \frac{4}{\ln 2} + \frac{8}{3} - \frac{1}{\ln 2} - 0$$

$$\boxed{= \frac{3}{\ln 2} + \frac{8}{3}}$$

$$= \frac{9 + 8\ln 2}{3\ln 2} = \frac{9 + \ln(256)}{\ln(8)}$$